CURRICULUM

QA
39.2
A 12
1976
Bk.1
Tch.Gd.

CURR

Ex libris
Universitatis
Albertaensis

QUAECUMQUE VERA

Digitized by the Internet Archive
in 2021 with funding from
University of Alberta Libraries

https://archive.org/details/amtappliedmathemdott_0

TEACHER'S GUIDE FOR AMT

AN INTRODUCTION

SECOND EDITION
SI METRIC

APPLIED MATHEMATICS FOR TODAY

DINO DOTTORI
GEORGE KNILL
ROBERT McVEAN
JOHN SEYMOUR

PREPARED BY JOHN SEYMOUR

THE RYERSON MATHEMATICS PROGRAMME

CURRICULUM
QA
39.2
A12
1976
bk.1
tch.gd.
CURR

Ex libris
Universitatis
Albertaensis

QUAECUMQUE VERA

Alberta Heritage Savings Trust Fund
Library Development Grant

TEACHER'S GUIDE FOR AMT

AN INTRODUCTION

SECOND EDITION
SI METRIC

PREPARED BY JOHN SEYMOUR

McGRAW-HILL RYERSON LIMITED

Toronto Montreal New York St. Louis San Francisco
Auckland Bogotá Guatemala Hamburg Johannesburg
Lisbon London Madrid Mexico New Delhi Panama Paris
San Juan São Paulo Singapore Sydney Tokyo

TEACHER'S GUIDE for AMT Introduction, Second Edition

Copyright © McGraw-Hill Ryerson Limited, 1980.
All rights reserved. No part of this publication may be reproduced,
stored in a retrieval system, or transmitted in any form or by any means,
electronic, mechanical, photocopying, recording or otherwise,
without prior written permission of McGraw-Hill Ryerson Limited.

ISBN 0-07-092436-8

1 2 3 4 5 6 7 8 9 MP 9 8 7 6 5 4 3 2 1 0

Printed and bound in Canada

LIBRARY
UNIVERSITY OF ALBERTA

Introduction

The Review and Preview to each chapter provides exercises to review skills that will be required for the coming work plus some additional material. Where skills are particularly pertinent it has been noted in this guide. It is not expected that every student need do all the Review Preview exercises.

The General Comments to each chapter give a suggested time allowance. The first figure allows not more than 2 periods for the Review Preview section. 1 period for testing and little extra time for review. These figures total 178 periods. The second figure assumes that topics will have to be retaught from time to time and more time is allowed for review. These figures total 216 periods. If your class is requiring more than the lower suggested time figure some of the optional topics.

The Section Comments are written in four parts.

A - The objectives.

B - A short opening exercise reviewing basic skills to be required by this section.

C - Comments and suggestions on teaching the topic.

D - Comments and suggestions on the exercise.

Where two or more sections deal with the same topic they have been covered together.

Chapter tests, Notes and Answers are provided to give examples of possible test questions. The tests should be modified to reflect the areas of concentration selected by the teacher.

Review Preview Chapter 1

Decimal fractions and problems

- In the 'test' section of this manual there is a pretest that may be used to determine where your students are with respect to decimal operations and percent. Exercises 1, 2, and 4 may be used for additional practice in the areas of weakness.

- Exercise 3 is suitable for enrichment for those students who do not need work in exercises 1, 2, or 4.

Chapter 1 NUMBER APPLICATIONS

General Comments

This chapter should be covered in 14 - 17 periods including Review Preview and testing. If deficiencies in the students skills with basic operations, order of operations and exponents require more time, then use periodic review periods rather than holding up progress at this point. This time frame is dependant on the use of calculators.

1.1 COMPUTATIONS

A Objectives: Use a calculator for simple calculations with the basic operations.

Use a calculator to calculate weekly or monthly gross pay given hours and rate.

B Just for fun a calculator riddle -- 'Why do you do what the principal tells you to?' Ans. 625 + 479 + 251 459

C Calculator Policy:

If the objective is to teach computation, then a calculator should not be used. If the objective is to solve problems then the use of a calculator can be considered appropriate.

D The exercise involves the calculation of wages from times and hourly rates and the students may have job related experiences. Class generated data would be valuable here.

1.2 ORDER OF OPERATIONS

A Objective: Evaluate an expression involving whole numbers and decimal fractions with basic operations complicated by brackets and exponents.

B Insert the operations signs x, ÷, +, - with the numbers 1▨ 2▨ 3▨ 4▨ 6 to make the greatest number; no brackets, do not change the order of the numbers, use each sign only once. Remove the above restrictions one at a time and note how the value may be increased.

C Give the students verbal instructions and have the questions written down so that the meaning is clear.

Example Three is added to 4 and the sum is multiplied by 5, becomes $(4 + 3) \times 5$

The product of 8 and 3 is subtracted from the product of 9 and 5 and the result is squared becomes $(9 \times 5 - 8 \times 3)^2$

Why is $((9 \times 5) - (8 \times 3))^2$ unnecessarily complicated?

D The examples are stretched out to be clear. Students should not be required to duplicate these steps in the exercises.

1.3 ROUNDING

1.4 ESTIMATING

A Objectives: Round a number to any assigned degree of accuracy.

Round off the numbers in a calculation and come to an approximation of the answer to ± 50% in most cases.

Gradually come to an appreciation of the accuracy required in a practical situation.

B How many basketballs could be placed in this room? After a few wild guesses are put on the board ask: How many basketballs wide is this room? more than 10? less than 50? How many basketballs long? High? Estimate the answer for capacity.

C When ever we purchase a number of items we should form in our minds an estimate of the total cost to be sure we are not embarrassed by lack of funds or penalized by a clerks error. A shopping list of 5 or 6 items might be put on the board for the students to quickly estimate the total cost, or expose the amounts briefly on bristol board.

D Exercise 1 - 4 offers a good opportunity to practice calculator skills. Set up a table.

Estimated answer	Calculator answer

1.5 EXPONENTS

A Objectives: Express repeated multiplication using exponents.

Evaluate simple expressions with positive and negative integral exponents and zero exponents.

B Make four different numbers using a 4 and a 5 with no other marks. Place the numbers in order of magnitude (45, 54, 5^4, 4^5).

C This topic prepares for standard notation. Stress the way powers increase rapidly in size with increasing exponents. With a few deft strokes of the chalk (10^{100}) we can write a number that exceeds the number of grains of sand in the world. (Since the volume of the earth is 1.1 x 10^{30} mm^3, even if it were solid sand the number of grains could not exceed $1000/mm^3$ hence 10^{100} is far in excess of the possible number).

D Question 6 deals with expressions that will come up later in work on compound interest. It is not essential at this point.

1.6 SIGNIFICANT DIGITS AND STANDARD NOTATION

A Objectives: Classify data as approximate or exact.
Change the form of numbers to or from standard notation.
Determine the number of significant digits in a number.

B Zero race. Have the students write a 'one' followed by as many zeros as they can write in 15's. Then discuss the size of the number that they have written and have the number written as a power of 10. What advantages are obvious in this form?

C Most students at this stage are quite clear on why counting numbers are exact but are still not comfortable with the concept that any measure has a built in error. Depending on the needs of your students you may expand on this aspect of the section or use standard notation as a vehicle to review place values and multiplication by power of 10's.

D Questions 1 and 2 can be handled orally and can lead to an assignment requiring student supplied data.

Review Preview to Chapter 2

This section provides additional material for practice with the calculator. Exercise 1 margin piece gives the figures referred to in the table 1-F, 2-G, 3-D, 4-E, 5-B, 6-C, 7-H, 8-A and reviews basic measuring formulas.

Chapter 2 STATISTICS

General Comments

This chapter should be covered in 12 - 15 periods. The main objective is to help produce informed and critical consumers of statistics. It is strongly recommended that the students be encouraged to bring in examples of graphs and the use of statistics from the market place and the business world. Government publications, company reports, car advertisements and newspaper business sections are all good sources of contemporary, meaningful data.

2.1 READING AND CONSTRUCTING GRAPHS

A Objectives: Recognize the bar, line, circle and pictograph.

Select the appropriate type of graph to display a set of data and construct the graph.

Read data from a graph.

B Find 1% of 360.

Use this to find 5%, 10%, 25%, 36%, of 360.

(3.6, 18.0, 36.0, 90.0, 129.6).

C Questions asked in the body of the chapter may be discussed orally as questions in the exercise.

Ask each student to check newspapers and magazines for examples of graphs. The evaluation of these graphs can lead to discussions of honesty in advertising.

D The exercise covers the aspects of displaying data and reading displayed data. Both are valuable but for the average student the second is more practical. Make use of student supplied graphs to practice extracting information. In question 2(d) the no moon planets are Mercury, Venus, Pluto. An interesting graph results if these planets are added to the list in the order they occur as we move out from the sun.

2.2 MEAN MEDIAN OR MODE

2.3 MEAN MEDIAN OR MODE

A Objectives: Find the mean, median and mode for a set of data.

Select a suitable measure of central tendency for a particular use.

B Select the average of the numbers 13, 16, 18, 23, 25 from the set (12, 15, 19, 23, 26) by estimation. This leads to a discussion of why 12 and 26 can be eliminated for certain and why 19 is more likely than 15 or 23. (The mean must be between the greatest and least values, the median is usually a good estimate of the mean.)

C The use of averages in sports such as batting average, yards gained per carry, goals against can provide an interest catching introduction. Have the students bring in a sports section from the newspaper or make an overhead transparency from a suitable article.

D The salaries of the executives in example 1 and question 6 of exercise 2-3 are a bit contrived. The purpose is to show how the mean is affected by one or two extreme values. A similar situation might be discussed where the class is given a test and two students are caught cheating and are given zero. Compare how the change in two marks from 70% and 40% to 0% and 0% changes the mean and the median of a set of class data.

2.4 THE HISTOGRAM

2.5 THE CUMULATIVE FREQUENCY CURVE

A Objectives: Organize data in a frequency table when given the class interval.

Use the frequency table to construct a histogram.

Distinguish between a histogram and bar graph in form and use.

Construct a cumulative frequency curve.

B Divide the whole numbers from 30 to 99 into 7 equal groups giving the largest and smallest number in each group. (The answer to this problem is used in example 1 of section 2.4).

C Stress the value of a neat frequency table, most errors occur in counting. To save writing some students will want to put only the boundary values on the horizontal axis. This often causes confusion and should be discouraged.

D Some of the questions requiring the construction of graphs lend themselves to group work with the graphs constructed on large sheets.

2.6 PERCENTILES

A Objectives: Explain the meaning of a percentile rank.

Calculate the percentile rank for any value from a supplied set of data.

B Turn the following fractions into percents.

$\frac{58}{100}$, $\frac{12}{25}$, 35 of 50, 18 of 30, 45 of 300

(58%, 48%, 70%, 60% 15%)

C Although few students will ever need to calculate a percentile rank, most will receive some test results in this form. Being able to calculate percentiles will increase their ability to internalize the results.

$$\text{Percentile } (x) \quad \frac{\text{number of values equal to or lower than } x}{\text{total number of values}} \times 100$$

D In the exercise the term percent for marks out of 100 has not been used to avoid confusion with the term percentile. With some students a discussion of how 60% = 80th percentile might be profitable.

Review Preview to Chapter 3

Chapter 3 requires that the students work with common fractions when investigating the nature of rational numbers, and with decimals when evaluating irrational expressions. These exercises give practice in those skills. The work on common fractions may be omitted without losing continuity in the rest of the chapter.

Chapter 3 RATIONAL AND IRRATIONAL NUMBERS

General Comments

This chapter should be covered in 12 - 15 periods.

The chapter reviews the conversion of decimals and common fractions and it touches on the concept of what an irrational number is. It then develops skills in handling expressions with radicals, approximating decimal values for radical expressions and gives practice with applications.

3.1 RATIONAL NUMBERS AS DECIMALS

A Objectives: Change any positive or negative common fraction to decimal form.

Change any terminating or repeating decimal to common fraction form.

Give the period and length of period for any repeating decimal.

B Find a number between each of the following pairs of numbers: 1.7 and 1.8, 4.90 and 4.91, 3.964 and 3.965, 0.00314 and 0.00315, 26.56 and 26.56321.

Fill in the table. Each number must be less than the one above it but greater than 7.6. Take turns and make the next move as difficult as possible.

7.6	7.7
7.6	
7.6	
7.6	
7.6	

Can you have two decimal numbers with no possible number between them?

C Students from different backgrounds may be used to different notation for repeating decimals. ie. 0.362, $0.\overline{362}$. Note the rule for changing from repeating decimals to common fractions - if one digit repeats multiply 10

if two digits repeat multiply by 100

if three ....

.

.

.

.

This topic affords the first real work with algebra for this year; use it to set up the types of solutions you will expect from here on.

D Question 7 could be done by changing the decimal fractions to common fractions but in a metric calculator age the direction of change should be to decimals.

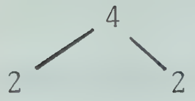
3.2 IRRATIONAL NUMBERS

A Objectives: Write rational and irrational decimal expressions.

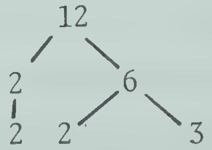
Determine if the square root of a whole number is rational or irrational by factoring.

Find the rational square root of a whole number by factoring.

B



and



are examples of factor trees. Complete a factor tree for 72, 120, and 760.

C Put exphasis on the fact that although $6 \times 6 = 36$ and $-6 \times -6 = 36$, $\sqrt{36} = 6$ and $\sqrt{36} \neq -6$ by definition. We are also laying the basis for the relation $\sqrt{ab} = \sqrt{a}\,.\sqrt{b}$, $a, b \geq 0$.

$$\text{ie} \sqrt{5 \times 5 \times 7 \times 7} = \sqrt{5 \times 5}\,.\,\sqrt{7 \times 7}$$

$$= 5 \times 7$$

But for now drill $\sqrt{5 \times 5} = 5$, $\sqrt{7 \times 7} = 7$ .. $\sqrt{a\,.\,a} = a$, $a \geq 0$

D Question 7 gives π to 9 places of decimals. The ancient Egyptians used an equivalent to 256/81 for π. Ask how close they came.

3.3 RADICALS PART I

3.4 RADICALS PART II

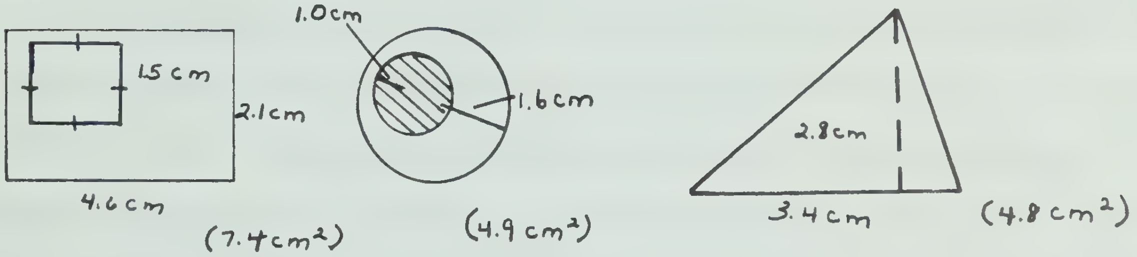
A Objectives: Multiply two radicals, mixed or entire.

Simplify a radical expression with one term to its simplest form.

Express a mixed radical as an entire radical.

Simplify radical expressions involving addition and subtraction which require that the radicals be simplified first.

B Find the area of the following figures



C Students having difficulty may be helped by returning to the ideas of the previous section and simplifying the radicals by prime factoring.

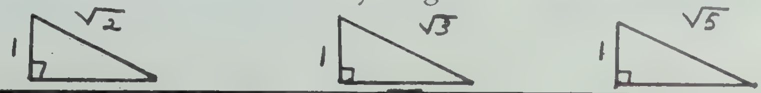
$$\sqrt{72} = \sqrt{(2 \times 2) \times 2 \times (3 \times 3)}$$
$$= 2 \times 3\sqrt{2}$$
$$= 6\sqrt{2}$$

Draw parallels between the work with radicals and the work with algebraic quantities.

$$3a \times 2b = 6ab \Leftrightarrow 3\sqrt{5} \times 2\sqrt{7} = 6\sqrt{5} \times \sqrt{7}$$
$$= 6\sqrt{35}$$

$$3a + 2a = 5a \Leftrightarrow 3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5}$$

D The last questions in exercises 3-3 and 3-4 review the ideas of area and perimeter. These can be omitted without losing continuity in the chapter. As enrichment for a good group of students you might have them investigate methods of drawing lines $3\sqrt{2}$ cm, $2\sqrt{3}$ cm, $\sqrt{5}$ cm long and constructing a triangle. One possible method serves as an introduction to the Pythagorean relation which will be covered in section 3.7.



3.5 - 3.6 SQUARE ROOTS PART I AND II

A Objectives: Estimate the square root of any number between 1 and 100 to the nearest whole number.

Estimate the square root of any number he is likely to experience to within one order of magnitude or better.

Use tables to find the square root of a number or to improve the accuracy of an estimate. (optional)

B Write the numbers 420, 6000, 0.71, 0.003 in scientific notation.

Use scientific notation to estimate the following quantities: 420 x 6000, 6000 x 0.003, 0.71 - 0.003 (2.5×10^6) (1.8×10) (2.4×10^2)

C Section 3.5 should be treated as an opportunity for the students to use knowledge they possess (the roots of the perfect squares from 1 to 100) to achieve surprisingly accurate results in estimating, as opposed to guessing. The coverage given section 3.6 should depend on the use your class has made of calculators. If students are in the habit of finding square roots of numbers by pushing a button they will probably resent having to look through tables. Use exercise 3-6 as a calculator exercise.

D When estimating, immediate feed back helps improve the accuracy. It is suggested that the students work in pairs.

Review Preview to Chapter 4

These exercises review basic skills of algebra that the student will use in chapter 4. Exercises 1 and 2 in particular should be covered.

Chapter 4 OPERATIONS WITH POLYNOMIALS

General Comments

This chapter should be covered in 18 - 22 periods. These are the basic skills of algebra that will be required by students going on to post secondary education in technology. Time will be required to establish a good grounding. However, if your students are showing signs of algebra fatigue before the end of the chapter, sections 4.11, 4.12, 4.13 could be postponed. Chapter 5 on graphing gives an opportunity for a more activity oriented interlude.

4.1 EXPONENTS IN MULTIPLICATION AND DIVISION

4.2 POWER LAWS

A Objectives: Write a power in expanded form or in exponential form.

Perform multiplication or division with monomial powers where all exponents in the question or answer are positive whole numbers.

Raise a power to a power (ie. $(2a^3)4$)

Distinguish between ab^c and $(ab)^c$ or $\frac{a^c}{b}$ and $\left(\frac{a}{b}\right)^c$

B Find the values for 5^6 and 6^5, which is greater? For any two consecutive numbers will the lower to the higher power always be greater? Try 2^3 and 3^2?

Complete the table:

1^2		2^1
2^3	$<$	3^2
3^4		4^3
4^5		5^4
5^6	$>$	6^5

Where does the change occur? Make a hypothesis and test it on larger values.

C Remind the students that if we have a single number such as 2 as a term it is not required to write 1 x 2 nor 2^1 similarly $a = 1a^1$.

In example 1(a) $5x^3 \times 3x^4 = 15x^7$ the perennial question is, 'How come its 5 x 3 but, 3 + 4, aren't we still multiplying?' It may help to expand the question. $5x^5 \times 3x^4 = 5 \cdot x \cdot x \cdot x \cdot 3 \cdot x \cdot x \cdot x \cdot x$

Then ask the student to simplify.

D The exercises are straight forward. It is suggested that you do an example such as:

$\left(\frac{8x^5y^6}{4x^2y}\right)^4$ and show that although you can expand first, the question is much simpler if the fraction is simplified first.

4.3 ZERO AND NEGATIVE EXPONENTS

A Objectives: Evaluate numerical expressions with zero or negative exponents.

Simplify algebraic expressions involving zero or negative exponents.

B Simplify the following expressions:

7 - 9, 2 - 4, 5 - 9, 4 - 8, 3 - 5, 3 - (-2), 5 - (-4), -4 - 2, -3 -6, -2 - (-1), 0 - 3, 4 - 0, -3 +0, 0 - (-4), 0 + (-2)

C Mastery of this topic is not required for further work in the chapter. Treat it as an introduction to work that will be covered again and expanded upon in later years.

D The exercise is straight forward up to question 6. The last question requires the student to put some ideas together such as the reciprocal of a reciprocal and adding powers instead of multiplying or dividing.

4.4 EVALUATING POLYNOMIALS

A Objective: Evaluate a polynomial of up to 4 terms given integer values for the variables.

B Complete the table:

a	b	a^2	b^2	ab	3ab	a^2b	ab^2	a^2b^2
2	4	(4)	(16)	(8)	(24)	(16)	(32)	(64)
3	-5	(9)	(25)	(-15)	(-45)	(-45)	(75)	(225)
-2	-4	(4)	(25)	(-10)	(-30)	(20)	(-50)	(100)
-3	-4	(9)	(16)	(12)	(36)	(-36)	(-48)	(144)

C If a preliminary exercise such as suggested in B has been done the students can quickly make up their own polynomials and simplify them, and later supply their own values for the variables with a calculator to check their answers.

3.7 APPLICATIONS OF SQUARE ROOT

A Objectives: Use the Pythagorean relation to calculate distances.

Evaluate expressions involving radical signs.

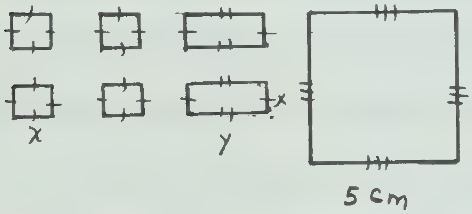
B What property do the triples of numbers (3, 4, 5) and (5, 12, 13) have in common? (It is assumed that by grade 10 these numbers will be old friends. $3^2 + 4^2 = 5^2$, $5^2 + 12^2 = 13^2$) Use these to make up other triples with the same property.

$$\begin{pmatrix} 3 & 4 & 5 \\ 6 & 8 & 10 \end{pmatrix} \quad \begin{pmatrix} 5 & 12 & 13 \\ 10 & 24 & 26 \end{pmatrix}$$

etc. etc.

C Students involved in technology courses may be able to supply more formulas using radicals or different applications of the Pythagorean relation. Pay a bounty for students bringing in new and different applications.

D In question 2(b) students might draw plans for the foundation of their own home or cottage and calculate the diagonal distances and draw a plan.



A diagram such as this may be put on the board and have the students give an expression for the total area. Then give x and y values.

$(4x^2 + 2\ x^{\ 4} + 25)$

D The numerical values in the exercises have been kept low so that the student can concentrate on the algebraic ideas. Once these have been mastered the students might be challenged by using larger numbers. If calculators are available a discussion of how to evaluate terms most economically might follow leading to the use of the memory to accumulate the value of the polynomial.

ie. $3a^2 \quad a^2 \times 3$

$4a^2b \quad a^2 \times 4 \times b$

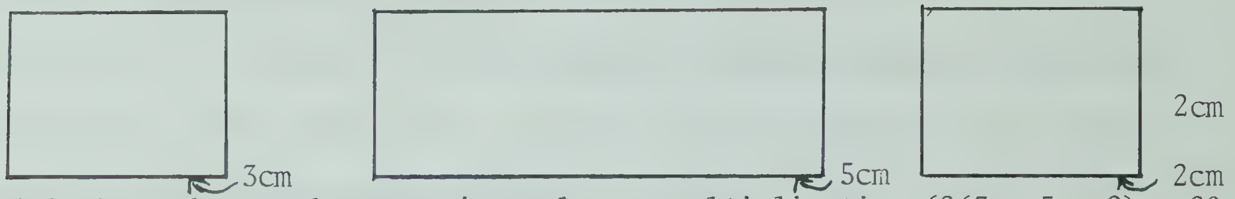
$2a^2b^2 \quad (a \times b)^2 \times 2$

4.5 ADDITION AND SUBTRACTION OF POLYNOMIALS

A Objectives: Use the distributive law to expand terms in a polynomial

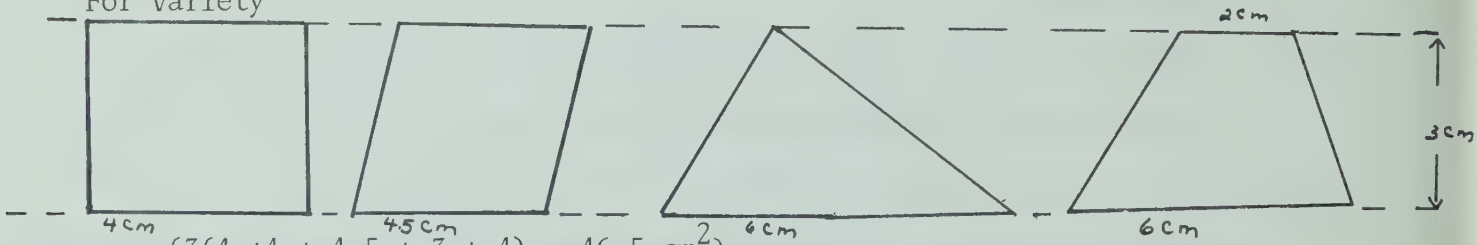
Reduce the resulting polynomial to its simplest terms.

B



Calculate the total area using only one multiplication $(2(3 + 5 + 2) = 20 \text{ cm}^2$.

For variety



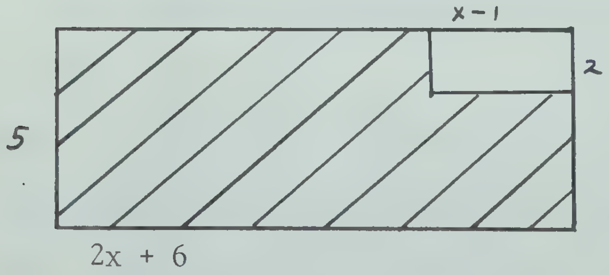
$(3(4 + 4 + 4.5 + 3 + 4) = 46.5 \text{ cm}^2)$

C With some students inserting an extra step in the first few examples helps: $2(x - 3) - 5(x + 7)$

$- 2(x - 3) + (-5)(x + 7)$

D Use this exercise to establish the format you want for the succeeding work with polynomials.

For students who finish early a lead on from question 6 might be of this type



$(5(2x + 6) - 2(x - 1))$

Find the shaded area.

4.6 MULTIPLICATION OF POLYNOMIALS

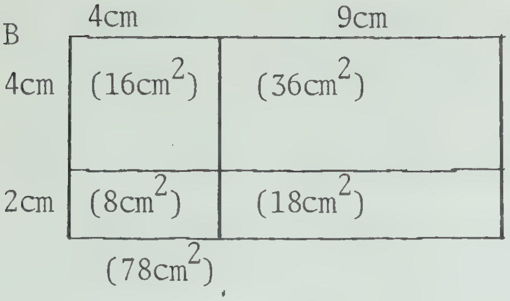
4.7 SQUARE OF A BINOMIAL

A Objectives: Find the product of two binomials.

Find products involving trinomials. (optional)

Square a binomial.

B



Find the area of each section of the figure. Find the total area.

C By stressing the rule in its general form, 'multiply each term in the first factor by each term in the second factor', the student can apply it to polynomials of any order.

The squaring of binomials should be treated as a further example of the product of two binomials. It is suggested that short cuts be saved until later.

D Example 3 and question 6 of section 4.6 give your more able and faster working students a chance to generalize and test their thinking. Question 5 of section 4.7 leads into section 4.8. If students are having difficulty this question might be handled as part of the next section.

4.8 SIMPLIFICATION OF POLYNOMIALS

A Objectives: Find the product of two or more pairs of binomials in an expression and simplify the resulting sum or difference of products.

Find the product of two binomial factors and a constant factor in a polynomial expression.

B Find the product (16.3 x 25) x 4.0

16.3 x (25 x 4.0)

(1630)

Does the answer change?

Which order makes a simpler calculation.

C If question 5 of section 4.7 was handled well this section will require very little teaching. The new points will be the introduction of a third constant factor in some products and the subtraction of some products. If question 5 was left because of student difficulties then start with the idea of a third constant factor and cover questions 1, 2 in exercise 4-8 then as a separate lesson complete questions 3, 4 and 5 in exercise 4-8 and question 5 in exercise 4-7 as time and abilities permit.

D See above.

4.9 DIVISION OF A POLYNOMIAL BY A MONOMIAL

4.10 DIVISION OF A POLYNOMIAL BY A BINOMIAL

A Objectives: Recognize the use of fractional notation or the use of a division sign as means of indicating division.

Divide a polynomial by a monomial where the divisor is a factor of the dividend.

Divide a polynomial of up to four terms by a binomial where the divisor is a factor of the dividend.

Rearrange terms and accommodate zero coefficient terms where necessary.

B First divide then add First add then divide

$6\overline{)12 + 24 + 54}$ $6\overline{)12 + 24 + 54}$ (15)

compare your answers.

C When discussing questions such as $\frac{3ab + 6bc}{3b}$ it is helpful to draw a parallel with the distributive property if $3b(3ab + 6bc)$ means multiply every term in the brackets by $3b$ then $\frac{3ab + 6bc}{3b}$ means divide every term over the 'bracket' by $3b$. The term bracket here, although not technically correct, makes sense.

In section 4.10 the close parallel between algebraic and numerical division suggests that some time spent initially reviewing the steps of long division would be worthwhile.

D In exercise 4-10 it is not necessary to allow for zero coefficient terms until question 4. With weak students you may wish to handle this idea separately.

4.11 FACTORING - THE COMMON FACTOR

4.12 FACTORING - THE DIFFERENCE OF SQUARES

A Objectives: Recognize when an expression is in factored form.

Factor polynomials with monomial and binomial common factors.

Factor a simple difference of squares.

B Find the highest common factor (HCF) for the following sets of numbers:

(4, 6, 10), (12, 24, 18), (10, 5, 25), (8, 20, 12),

(15, 17, 31), (21, 14, 28), $\underset{(x)}{(x^2, xy)}$, $\underset{(ab)}{(a^2b, ab^2)}$, $\underset{(xy)}{(xy, xyz)}$

C There is a fair bit of mathematics involved in finding the common factor 'by inspection', however most students do it amazingly well. Time spent in multiplying out to check the accuracy pays big dividends in proficiency later.

For the student who does not see the common factors try expanding some expressions.

$$
\begin{aligned}
12a^2b + 18ab^2 &= \underline{2} \times 2 \times \underline{3} \times \underline{a} \times a \times \underline{b} + \underline{2} \times \underline{3} \times \underline{3} \times \underline{a} \times b \times \underline{b} \\
&= 2 \times 3 \times a(2a + 3b) \\
&= 6a(2a + 3b)
\end{aligned}
$$

The difference of squares algorithm is quickly memorized and is proven by expansion.

D If the students have difficulty with the binomial common factors some will be helped by substituting. ie. $x(x + 2) + 3(x + 2) = xa + 3a \ (a = x + 2)$

$$
\begin{aligned}
&= a(x + 3) \\
&= (x + 2)(x + 3)
\end{aligned}
$$

4.13 FACTORING - TRINOMIALS

A Objectives: Factor a trinomial when the coefficient of the squared term is 1.

B The Greeks were interested in the properties of numbers. If a number was equal to the sum of its factors including 1 but not the number, it was called a perfect number. 6 is a perfect number since 1 + 2 + 3 = 6 496 is also a perfect number.

Between 6 and 496 is one other perfect number. Can you find it? It is less than 30. (28)

C This section has been set out in the text to facilitate a step by step approach discussing questions 1, 2 and 3 orally.

D Question 8 involves common factors. This was done to give further challenge to the better student. It is not expected that most students at this level will spot common factors in other factor types without a hint being supplied.

Review Preview to Chapter 5

Exercise 1 introduces function notation, it should be treated as enrichment. Exercises 2 and 3 review graphing on the number line as an introduction to graphing in the plane to be covered in chapter 5.

Chapter 5 FUNCTIONS AND RELATIONS

General Comments

This chapter should be covered in 18-22 periods. There are several open ended topics such as interpolation and extrapolation, graphing non linear relations and inequalities; their treatment will affect the amount of time taken.

If you are not stressing this topic sections 5.1 to 5.6 may be considered core.

5.1 ORDERED PAIRS

5.2 GRAPHING SETS OF ORDERED PAIRS

A Objectives: Give the domain and range of a binary relation presented as a set of ordered pairs.

Find the range of a relation given the domain and defining statement.

Set up labelled axes and scales and plot a set of ordered pairs.

B Find my rule game. A student gives a set of ordered pairs and other students guess the rule. When an error is made another set of ordered pairs is given using the same rule.

Example: (2.6) guess 'times 3', wrong
(5,12) guess 'times 2 plus 2, correct

The student who guesses the rule makes up the next rule.

C The names, 'relation','domain', and 'range' are handy words to use. If they are used consistently in class the students will use them spontaneously. The graphing of points should be review. Use these lessons to set up the format you want used with your graphs.

D In exercise 5-2 question 4-13 involve word descriptions of relations. If the class is moving along well, take time to have the students save time by writing them on the graphs in mathematical shorthand. '...where the second component is twice the first.' becomes y=2x.

5.3 GRAPHING RELATIONS PART 1

5.4 GRAPHING RELATIONS PART 11

A Objectives: Given a relation and a domain, complete a table of values and plot the graph. The relations covered will all be functions with the exception of those in the form x=a.

Select between a set of discrete points or a solid line for the graph according to the stated domain.

Recognize a relation which is also a function (optional).

B From the following sets of ordered pairs, find a pattern and write the next two elements in the sequence.

(1,2), (2,5) (3,7).......	(4,9), (5,11)
(1,5), (3,8), (5,11).....	(7, 14), (9,17)
(0,0), (1,1), (2,4)......	(3,9), (4,16)
(1,3), (2,3), (3,3)......	(4,3), (5,3)

C Stress 'If the ordered pair satisfies the equation the point is on the graph.' Once the mechanics of finding values for y and plotting points have been mastered, stress the search for patterns in the numbers and patterns in the points. When progressing from point graphs to line graphs it is worth discussing situations where point graphs better describe the real life situation, for example unit cost. Ketchup is bought by the bottle and the cost of 2½ bottles is not meaningful data.

In section 5.5 the idea of function should be discussed in conjunction with question 1, but not stressed. If the students are not clear on the other points don't further complicate the topic.

D In exercise 5-4 question 5 various forms of notation are used to widen the students experience. With insecure students a preliminary word of explanation is in order. Also note one quadratic function at the end of the question.

In exercise 5-5 question 6, the graphs are all vertical or horizontal lines. The graphs are not difficult but the discussion of domain and range may cause confusion between the range of the variable and the range of relations.

5.5 INTERPOLATING AND EXTRAPOLATING

A Objectives: Take a partial set of data, graph the known values and draw a reasonable graph between and beyond the points.

Interpolate and extrapolate values from the graph.

Explain why the results so obtained may be in error.

B If a hen and a half lays an egg and a half in a day and a half how long does it take one hen to lay 1 dozen eggs? (18 days).
Two hens to lay 1 dozen eggs? (9 days). Six Hens? Twelve hens?
Eighteen hens? Thirty-six hens?......? (Starts to lose meaning).

If three men can build a house in 90 days how long will it take six men? 10 men? 90 men? (34, 9, 1) (Not realistic). If one student can walk from Newmarket to Aurora in 45 minutes how long will it take two students? (45 minutes).

C The above examples are intended to illustrate the pitfalls of extrapolation, firstly the laws of diminishing returns, sooner or later bigger and more just get in each others way, and secondly some data is not suitable for extending. This topic invites a lot of discussion.

D Exercise 5-5 question 4 is suitable for group work on large graph sheets. The counting of similar triangles can lead to some heated discussions. Better to omit this question than to rush it.

5.6 $y = mx + b$

A Objectives: Transform a linear equation in x and y to the form $y=mx+b$ and give the values of m and b.

Identify equations of parallel lines from value of m.

Determine the y-intercept of a line from its equation.

Describe the general direction of a line from its slope.
i.e. up to the right, up to the left. (optional).

B Solve for y: $3y = 5$ (5/3), $\frac{1}{4}y = 5$ (20)
$y - 4 = 8$ (12) $2y + 6 = 4$ (-1) $3y - 2 = 7$ (3)

C The purpose of this section is to introduce the idea of parameters and have the student realize that if an equation is put in a special form certain information can be read from it. Using the idea of decoding a message might help, the information is there in the equation, we just need to be able to read it.

Investigation 5-1 is suitable for group work on large graph sheets. From it the students should learn that the value for m controls the direction of the line and the value for b controls where the line crosses the y axis.

D Exercise 5-6 practices the skills of transforming equations. The principal objective of the chapter is the geometry. It is sufficient that the students recognize that any linear equation in x and y can be put in the $y = mx + b$ form. For this reason the equations in the investigation are given in the $y = mx + b$ form.

5.7 GRAPHING RELATIONS USING THE INTERCEPTS

A Objectives: Determine the x and y intercept of a line from the graph.
Determine the x and y intercept of a line from its equation.
Graph a line given its x and y intercepts.

B Solve the following pairs of equations:

$2x = 6$, $3y = 6$ (3,2) $5x = 20$, $4y = 20$ (4,5)
$6x = 0$, $7y = 0$ (0,0) $8x = 12$, $9y = 12$ (3/2,4/3)

C Review the idea that if the ordered pair satisfies the equation the point is on the graph.
Any number can be put into the equation.
i.e. $3x + 2y = 6$ if $x=1$, $3 + 2y = 6$, $y = 3/2$, (1, 3/2) on graph.
if $x=6$, $18 + 2y + 6$, $y = -6$, (6, -6) on graph.
if $y = 2$, $3x + 4 = 6$, $x + 2/3$, (2/3, 2) on graph.

The easiest value to substitute in is zero, hence the use of intercepts.

D In the exercise we have not used equations of lines that pass through the origin. With good students it would be profitable to discuss how we can tell from the equation when a line will pass through the origin $(b=0)$ and how in that case it is necessary to find a second ordered pair to satisfy the equation if we wish to graph it.

5.8 POSSIBLE VALUES OF A RELATION

5.9 GRAPHING INEQUALITIES 1

5.10 GRAPHING INEQUALITIES 11

A Objectives: Test an ordered pair and determine if it satisfies a relation which may be an equation or inequations.

Recognize that if the ordered pair satisfies the relation it represents a point of the graph of the relation.

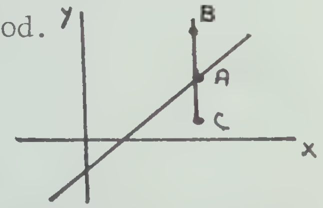
Select a test point and find the area where $y > mx + b$ and $y < mx + b$.

Graph the inequations of the form $y > mx + b$, $y < mx + b$, $y \leq mx + b$, $y \geq mx + b$,

B Write three ordered pairs that satisfy each of the following relations; $y = x$, $y > x$, $y < x$.

C The sequencing of the skills involved in this topic is critical. If the student cannot tell if the numbers satisfy the relation he will not be able to determine where to graph it. Minimum expectation here should be that the student realizes that for all points on one side of the line $y > mx + b$, for points on the line $y = mx + b$ and for all points on the other side $y < mx + b$.

The test point method gives valuable practice in the use of number skills, however it might be supplemeted by the vertical line method. Since the x coordinate is the same for A, B and C, if at A, $y = mx + b$ then at B, $y > mx + b$ and at C, $y < mx + b$. One drawback is the difficulty in applying it to the form $ax + by > c$ or $< c$.



D Exercise 5-9 question 7, 8 and 9 have equations in different form. It should be acceptable to represent the areas of the graph by first putting the equation in the $y = mx + b$ form. Better students should be encouraged to determine where $2x + y > 3$ etc. Exercise 5-10 question 3, questions of the form $x > a$ or $x < a$ seem to cause trouble. Some extra preliminary explanations may be necessary.

5.11 NON LINEAR RELATIONS

A Objectives: Recognize when the graph of a relation will not be a straight line.

Graph some of the simpler quadratic relations.

B Find two solutions for each of the following equations;

$x^2 = 25\ (\pm 5)$ $y^2 + 16\ (\pm 4)$ $a^2 = 36\ (\pm 6)$ $b^2 = 49\ (\pm 7)$

$2x^2 = 8\ (\pm 2)$ $3y^2 = 3\ (\pm 1)$ $5a^2 = 45\ (\pm 3)$ $-b^2 = 81\ (\pm 9)$

C In this section we are widening the students mathematical experience. It provides good opportunity for the more able students to work ahead, in groups perhaps, while those with difficulties in the basic graphing are getting remedial help.

D If questions 1(a), (b), (d) are graphed on the same axis and then 1a) is repeated with 1c) the students may form some interesting hypothesis. Some may compare $y = x^2 + 2$, $y = x^2 - 2$, $y = x^2 - 3$ with the format $y = mx + b$.

Review Preview to Chapter 6

Exercise 1 reviews work of chapter 4 that will be required in this chapter. In Exercise 2 'by inspection' does not imply guess and test, but rather a questioning approach to the equation. ie. x + 3 = 12 is read, 'What number plus 3 equals 12?' Use it to reemphasize the idea that we have solved an equation when we have found the values for the variable that make the statement true.

Chapter 6 Equations

General Comments

This chapter should be covered in 18-22 periods. It covers solving first degree equations in one and two variables and problem solving. The emphasis is on moving from an intuitive approach to equation solving to a formal approach.

6.1 EQUATIONS IN ONE VARIABLE PART I

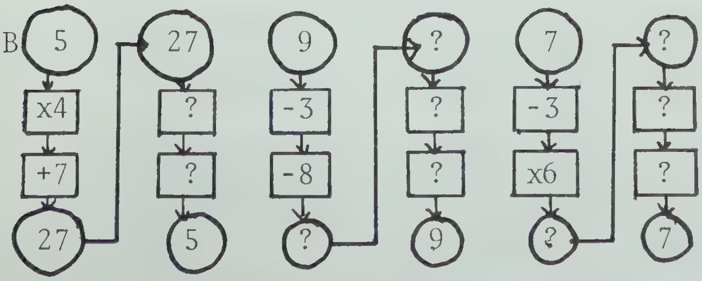
6.2 EQUATIONS IN ONE VARIABLE PART II

A Objectives: Use the axioms of addition, subtraction, multiplication and division to solve first degree equations.

Simplify both sides of an equation where required to enable the use of the axioms.

Use substitution to check results.

B



C The documentation in the examples is not intended as a model for the student solutions - it is there for explanation. Initially the students might be given a choice as to whether they prefer to write:

$x + 5 = 6$		$X + 5 = 6$	
$X + 5 - 5 = 6 - 5$	or	$X = 1$	(-5)
$X = 1$			

In section 6.2 take special note of example 3. After multiplying through to remove fractions there is a tendency to forget that we are subtracting the quantity $3x - 1$.

D No instructions have been given in the exercise concerning checking. It is suggested that a selection of questions be checked. In the more complicated equations the error is as likely to be in the checking as in the solution.

6.3 EQUATIONS INVOLVING PRODUCT OF POLYNOMIALS

A Objectives: Simplify polynomials involving products and solve the resulting equations, limited to cases where the simplified equation is of the first degree.

B Expand the following expressions:

$3(x - 1)$,	$(a - 3)(a + 1)$,	$(2a - 1)(a + 2)$,	$2(x + 3)(2X - 1)$
$(3x - 3)$	$(a^2 - 2a - 3)$	$(2a^2 + 3a - 2)$	$(4x^2 + 10x - 6)$

C Sections 6.1 and 6.3 involve the fusion of the skills of solving equations with the skills of simplifying polynomials. If the students have not mastered the simplification skills, for further practice, first have them treat the L.S. and R.S. of equations in questions 1 and 2 as separate simplification questions. After these have been checked or compared with a neighbours work for accuracy they complete the solution.

D Question 1 is a review of the type solved in section 6-1. A reasonable assignment would be questions 1, 2, and 3 for average math students and questions 2, 3 and 4 for the above average.

6.4 SOLVING EQUATIONS IN TWO VARIABLES GRAPHICALLY

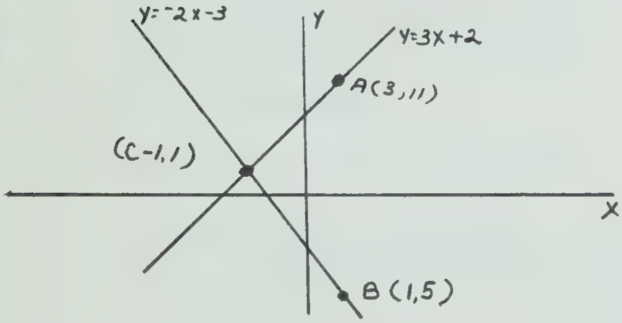
A Objectives: Explain why the co-ordinates of the points of intersection of two graphs must satisfy both relations.

Explain what is meant by the solution set of a system of simultaneous equations.

Graph and solve suitable pairs of linear equations.

B On the same set of axes graph $y = 3x + 2$ and $y = 2x - 3$.

C When the students have graphed the pair of relations in B the discussion can be developed from a chalkboard solution:

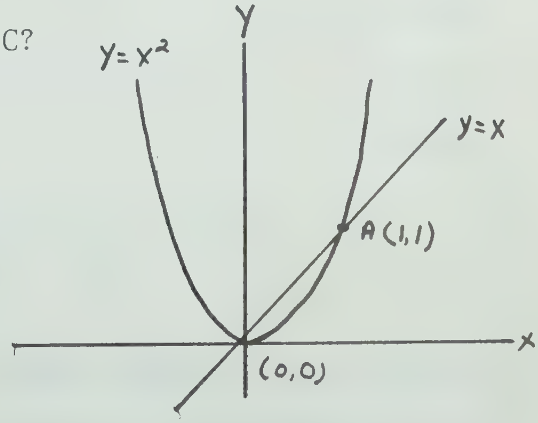


What must be true of the co-ordinates of A?

B?

C?

The ideas can be generalized further with the graph.



6.5 SOLVING EQUATIONS IN TWO VARIABLES BY COMPARISON

6.6 SOLVING EQUATIONS IN TWO VARIABLES BY ELIMINATION

A Objectives: Select a suitable method and solve a system of two linear equations in two variables algebraically.

B Solve

$2x + 1 = x + 3$, $\frac{x - 1}{2} = \frac{x + 2}{3}$, $\frac{1}{2}x + 3 = \frac{2x + 1}{3}$

($x = 2$) ($x = 7$) ($x = 16$)

C In our experience solving by comparison seems to make more sense to more students. For this reason it has been put first. Once it has been mastered the method of elimination can be shown to be much easier for some systems.

It is suggested that the students be told that on a test they may use which ever algebraic method they prefer. One method mastered is better than two methods confused.

D In exercise 6-6 it has been suggested in the margin that fractions be cleared before attempting a solution. This is a good rule for most students, however questions like 5(a) and (b) can be very neatly solved by first adding, then subtraction. This might be discussed with some of the more able students.

6.7 PROBLEM SOLVING

A Objectives: Apply the skills of solving equations to problem situations.

B If '$3x + 4 = 6$' becomes ' 3 times a number increased by 4 is 6 'read

$2x + 7 = 5$, $\frac{x + 3}{4} = 2$, $4x - 3 = 10$, $\frac{x}{3} - 1 = x + 4$.

C Questions 1 and 2 practise translating word expressions and word equations into algebraic statements and equations. A test can be set up with one student making up the word equations and another translating it.

A four step solution helps students organize their work in problem solving.

1. Represent the unknowns by variables (statements)
2. Form equations (translation)
3. Solve the equations
4. Answer the question (statement)

D One of the best aids to problem solving is problem creating. If the students can supplement or replace text problems by problems of their own devising the work will be more meaningful and better retained.

Review Preview to Chapter 7

These exercises provide a review of the most commonly used metric units of length, capacity and mass. The investigations in the next chapter require the use of measurement. If your students are well versed in metric units they might be allowed to omit the tables and go directly to the problems.

Chapter 7 Ratio and Proportion

General Comments

This chapter should be covered in 14 - 17 periods. Be prepared to collect necessary apparatus for the investigations in section 7.3.

7.1 RATIO

A Objectives: Compare two quantities as a ratio in its simplest form.
Simplify a given ratio.
Divide a given quantity into two or three parts in a given ratio.

B Simplify the following fractions

$\frac{10}{12}\ (\frac{5}{6}),\quad \frac{35}{40}\ (\frac{7}{8}),\quad \frac{25}{100}\ (\frac{1}{4}),\quad \frac{15}{45}\ (\frac{1}{3}),\quad \frac{51}{17}\ (\frac{3}{1})$

$\frac{1\frac{1}{2}}{3}\ (\frac{1}{2}),\quad \frac{\frac{3}{5}}{3}\ (\frac{1}{5}),\quad \frac{4}{\frac{1}{3}}\ (\frac{12}{1}),\quad \frac{4}{2\frac{1}{2}}\ (\frac{8}{5}),\quad \frac{\frac{5}{2}}{5}\ (\frac{1}{2}),$

C The student is often tempted to think of a ratio as just another form of fraction. For this reason we have quickly moved to the 3 term ratio so that distinction will have to be drawn. In example 2, it helps to put 2x:x on the board and give x several values to show that all the numbers so generated really are in the ratio of 2:1.

D Note questions 4, 5, 6, 10. These questions attempt to help the student discover important information about ratios. Some students may require preliminary discussion on these.

7.2 Proportions

A Objectives: Use the fact that if a:b = c:d

then $\frac{a}{b} = \frac{c}{d}$ and ad = bc

with real numbers.

Find a missing proportional.

Find the mean proportional between two numbers.

B Solve: $3x = 18$ (6), $5x + 35$ (7), $7a = 2 \times 14$ (4)

$4.5a = 9 \times 15$ (30), $4 \times 3 = 6a$ (2), $a^2 = 4 \times 9$ (6, -6).

C Of the properties of a proportion listed on page 144 only 1, 2 and 3 are required to do the questions of the exercise. Property 5 is used in Example 3 as an alternate solution. It should be discussed with able students.

D Questions 9 and 10 involve gears. This can be expanded on by having the students supply directions of rotation. Further applications involve pulleys and belts or sprocket wheels and chains as on bicycles. A project could involve listing the ratios available on a 10 speed bicycle.

7.3 INVESTIGATIONS IN VARIATION

A Objectives: To discover through experiment and analysis that the mathematics of variation describes naturally occuring relationships.

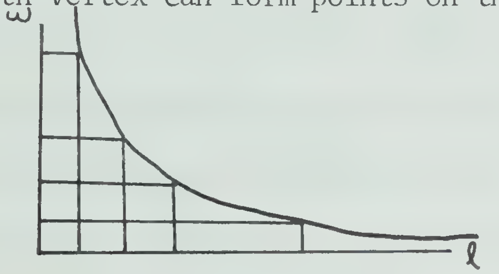
To discover that mathematical descriptions can be used to predict outcomes.

B Go directly to the investigations:

Investigation 7.1 Rectangles I $\propto \frac{1}{w}$

C Notes

By drawing the rectangles on graph paper with 2 sides along the axes the 4th vertex can form points on the graph.



Students note that ℓ x w = k (constant) and graph is curved and approaches the axes. A good discussion point after Chapter 6, will the graph have x and y intercepts? (no)

Investigation 7.2 Ramps I $t \propto l$

The measurement of time makes this a difficult investigation to get consistent data from. In free fall $t \propto \sqrt{d}$ however, if the slope is kept small friction and the horizontal component tend to make this relation linear. This investigation is susceptible to many pitfalls and probably will require a liberal interpretation for the curve of best fit to be a straight line.

Investigation 7.3 Ramps II

Not an example of variation. The curve varies with heights used.

Investigation 7.4 Rectangles II

This is not an example of variation but the curve is parabolic with equation $A = 50\ell - \ell^2$. The vertical axis on the graph should extend from 0 the 625 cm^2. The largest area will be generated by a square 25 cm x 25 cm.

Investigation 7.5 Projections I $h \propto d$

Note that the distances ‘d’ are measured from the light source not from the screen. Measuring from the screen makes an interesting variation which gives an example of partial variation. Good results are possible here with a little care.

Investigation 7.6 Projections II $h \propto \frac{1}{d}$

An interesting example of inverse variation with the graph truncated when d = distance to the screen. If the screen were replaced by acetate and the object allowed to pass beyond it, the height of the object as it appeared on the acetate would complete the graph.

Investigation 7.7 Water Levels I $d \propto w$

The biggest problem here is spillage accidental and otherwise. A good source of data.

Investigation 7.8 Water Levels II $h \propto \frac{1}{d}^{2}$

This is an example of inverse variation so the graph will have the usual form, asymptotic to the axes, but because the depth varies inversely as the square of the diameter the shape will be skewed to the ‘d’ axis.

7.4 DIRECT VARIATION

A Objectives: Given that $a \propto b$, write a variation equation and proportion.

Given a set of corresponding values, find b′ for any given value of 'a'.

Come to realize intuitively that with direct variation as 'a' increases, 'b' increases.

Illustrate the data graphically and recognize the form of the graph.

B Find the missing proportional, $x > 0$

$\frac{4}{6} = \frac{8}{x}$ (12), $\frac{9}{12} = \frac{3}{x}$ (4), $\frac{5}{7} = \frac{35}{x^{\text{[illegible]}}}$ (7), $\frac{7}{2} = \frac{28}{x^3}$ (2)

C Two methods of doing direct variation questions are shown and the one stressed should depend on the aid to calculation available. The proportion method is slide rule oriented while multiplication by a constant is calulator oriented.

If the variation constant can be put into memory then a table of values can be turned out very quickly. Students enjoy graphing by this technique.

D. In questions 3 (d), 5, 8, 10 the dependant variable varies as a power of the independent variable. These questions may require special mention.

7.5 INVERSE VARIATION

A. Objectives: Given that $a \propto \frac{1}{b}$ write a variation equation of the form $a_1b_1 = k$ and $a_1b_1 = a_2b_2$.

Given a set of corresponding values, find b for any given a.

Come to realize intuitively that with inverse variation as 'a' increases, 'b' decreases.

B Solve for b, $b > 0$

$3 \times 7 = 9b$ $\left(\frac{7}{3}\right)$, $4 \times 7 = 14b$, (2), $5 \times 17 = 20b$ $\left(\frac{17}{4}\right)$

$5 \times 4 = 20b^2$ (1), $16 \times 4 = 8b^3$ (2), $5 \times 10 = 2b^2$ (5).

C If direct variation has been mastered this section will require little teaching. Stress the idea of a constant product.

D Question 2 might lead to a discussion of types of jobs where this relation holds, ie. assembling small mechanisms in large numbers: and where it does not hold, a large job involving sequenced tasks. Question 3. pa = pascale, a pascale is the pressure exerted by a 1 newton force acting on 1 m^2. Normal atmospheric pressure at sea level is 100 kPa.

7.6 PARTIAL VARIATION (optional)

A Objectives: Given the values for the constants and the partial variation relation find pairs of corresponding values.

Illustrate the data graphically and recognize the form of the graph.

B Complete the table of values for the relation $y = 10x + 5$

x	4	6	7.5	9.7
y				

(45, 65, 80, 102)

C Partial variation is one of the most frequently occuring relations in establishing costs of operation - an automobile has fixed annual costs of depreciation, license, insurance plus variable costs of gas, oil and maintenance depending on the distance driven. A house has fixed costs of depreciation, taxes, insurance plus variable costs of heating, power, water, depending on the number of occupants. The cost of a reception would be partially fixed; hall rental and music; and partly dependent on the number of guests, food and refreshment. These and other examples can be used for student generated questions.

D Questions 3 and 4 involve equations $c = \frac{5n}{2} + 150$ and $V = 5.5 + 2.5t$
A brief review and comparison with $y = mx + b$ may assist in the graphing.

Review Preview to Chapter 8

Chapter 8 deals with personal finance. This exercise reviews the work with fractions and percentages involved in calculating interest.

Chapter 8 Personal Finance

General Comments

This chapter should be covered in 11 - 13 periods. Because of inflation and rapidly fluctuating interest rates it is suggested that considerable use of current data from advertisments and business sections be used to update the text material.

8.1 MANAGING SMALL FUNDS

A Objectives: Take on a position as treasurer of a club or society, or keep a record of the financial dealings of a small enterprise in a clear and business like manner.

B What are the elected positions on the executive of most clubs? (president, vice-president, treasurer, secretary). What are the main responsibilities of each?

C This section deals with taking over the job of treasurer of the students council from taking over signing authority on the bank account, and keeping a financial record to presenting a treasurer's report. It is intended to be preparation for the jobs that responsible citizens often fall heir to. A selection of bank forms, available at most banks without charge, will be helpful.

D The exercise is in several parts and practices the various skills discussed as they are taken up. It is suggested that they be done as they occur rather than all at once at the end of the chapter.

8.2 FINANCING A MAJOR PURCHASE

8.3 SIMPLE INTEREST

A Objectives: Recognize that certain purchases are likely to be made using credit.

Reasonably weigh the cost of credit against the need for an immediate purchase.

Through the calculation of interest and amount to be repaid, come to a further understanding of the cost of credit.

Given any three of the variables in the relation I = Prt calculate the third.

B Solve

a) a = 500 x 0.06 x 0.5 (15)　　b) 60 = a x 0.12 x 1 (500)

c) 200 = 1000 x a x 2 (0.1)　　d) 500 = 2000 x 0.1 x a (2.5)

C When students calculate the cost of borrowing they are better able to internalize its significance. Catalogues, applications for credit cards and mortgage information in house advertisements all give good sources for up to date borrowing costs. Over the period of updating this text interest rates have gone from 12% to 20% and back to 14%. Be current with your students.

D Exercise 8-2 question 2(c). Usually we think of a credit purchase involving a relatively large amount of money and the purchase of durable goods, something that can be repossessed if necessary. The advent of the credit card has made credit available for almost anything.

8.4 THE EFFECTIVE INTEREST RATE

8.5 SOURCES OF CREDIT

A Objectives Calculate the effective interest rate from the number and size of payments and the cash price.

Suggest various sources of credit.

Take up a schedule of payments for a credit purchase involving fixed payment or fixed payment on principal plus interest. (optional)

B Find the average of the following amounts.

a) \$436.50, \$27.50 (\$232) b) \$1395, \$250 (\$822.50)

c) \$5280, \$120 (\$2700).

C The consumer protection legislation which requires lenders to disclose the true rate of interest has removed some of the need for buyers to be able to calculate the true rate. However the technique of averaging the largest and smallest values of a series to find an approximation of an equivalent constant term series is worth investigating for mathematical reasons.

If your school teaches computer science or data processing, one of the students could programme a solution to one or more of the questions in 8.5. This would be a good opportunity for the students to investigate these courses.

D Exercise 8-4 question 6 gives an alternate method of finding true interest rate. It could be used instead of the average principal method. The calculation of payment schedule is rather long for inclusion on a test. It is suggested that one of the questions of exercise 8-5 be used as a hand in assignment.

8.6 PURCHASING AN AUTOMOBILE

A Objectives: Calculate the monthly cost of financing the purchase of an automobile.

B If a car uses gas at the rate of 9.4 L/100km and it is driven 15000 km per year how much gas will it use? At your local price of gas how much will this cost?

C The text has not investigated the cost of running a car. This would be a good project or class discussion topic.

D Inflation being what it is, the students might be asked to supply up to date vehicle prices.

Review Preview to Chapter 9

This exercise practices the skills of adding amounts of money and finding total charges where only some of the items are taxable. A practical business exercise for students who may work taking cash.

Chapter 9 Matrices

General Comments

This chapter should be covered in 9 - 11 periods. The topic is usually considered optional, it does have the value of allowing students with a limited background to start a new topic with easily mastered concepts. The arithmetic involved can be as simple or as complicated as the numbers the teacher wishes to choose to make up the matrices. Also the topics can be shown to have 'beyond school applications'.

9.1 INTRODUCTION TO MATRICES

A Objectives: Identify a matrix.

State the dimensions of a matrix.

Use and recognize the basic vocabulary of matrices, including such terms as dimension, rows, columns and elements.

Apply matrices to the storage of data.

B A magic square is an example of a matrix. If the rows and columns of this magic square add to 15, fill in the missing numbers:

$$\begin{pmatrix} 8 & 1 & (6) \\ (3) & 5 & (7) \\ (4) & (9) & (2) \end{pmatrix}$$

9.2 ADDITION OF MATRICES

9.3 SUBTRACTION OF MATRICES

9.4 SCALAR MULTIPLICATION

A Objectives: Recognize when addition and subtraction of matrices is possible and perform the operations.

Distinguish between matrices and scalars.

Perform scalar multiplications.

Use of matrix operations in simple applications.

B The magic square $\begin{pmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{pmatrix}$ totals 15, the magic square $\begin{pmatrix} 10 & 3 & 8 \\ 5 & 7 & 9 \\ 6 & 11 & 4 \end{pmatrix}$ totals 21. Use these two squares to form a square that totals 36.

$$\begin{pmatrix} 18 & 4 & 14 \\ 8 & 12 & 16 \\ 10 & 20 & 6 \end{pmatrix}$$

What property of a magic square does it not possess?
(It is not a sequence of natural numbers)

Double each term of the square Matrix $\begin{pmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{pmatrix}$

What happens to the totals of the rows and columns?

Predict what will happen to the total if each term were multiplied by 4?

C The students usually get on to the addition, subtraction and scalar multiplication of matrices very quickly. This is a good opportunity to practice them on the basic operations with integers, decimals and common fractions.

D Section 9.4 question 3 gives the students algebraic expressions to evaluate where the replacement values are matrices rather than scalar values. An extension of the questions could be the solution of a matrix equation of the type:

$$3A + \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 5 & -7 \\ 9 & -8 \end{pmatrix}$$

9.5 MULTIPLICATION OF MATRICES

A Objectives Recognize when multiplication of two matrices is possible.

Multiply matrices up to 3 x 3.

Apply multiplication of matrices to simple situations.

B Simplify the following expressions:

$3(4 + 6) + 2(7 + 5)$

$4(3 - 2 + 2) + 3(6 + 1 - 3)$

$10(4 + 8 - 3) - 5(3 - 2 + 8) + 7(1 + 0 - 4)$

C This is an introduction to a topic that can be expanded upon in future years. At this stage it is sufficient for the students to have their horizons broadened without demanding proficiency. A possible extension for more able students is an investigation of the lack of commutativity of matrix multiplication. $A \times B \neq B \times A$ and in some cases $B \times A$ is not possible.

D The exercise is of limited size consistent with the above intention. Further examples could be derived from stock taking in a chain of stores to find the value of the inventory or of sales.

$$\begin{array}{ll} & \begin{array}{ccc} \text{REFRIGERATORS} & \text{STOVES} & \text{TOASTERS} \end{array} \\ \begin{array}{ll} \text{Inventory} & \text{Store A} \\ \text{or sales} & \\ & \text{Store B} \end{array} & \left(\begin{array}{ccc} 8 & 12 & 25 \\ 6 & 9 & 21 \end{array}\right) \end{array} \begin{array}{ll} \text{COST} & \\ \left(\begin{array}{r} \$560 \\ \$325 \\ \$\ \ 35 \end{array}\right) & \begin{array}{l} \text{Refrigerat} \\ \text{Stoves} \\ \text{Toaster} \end{array} \end{array}$$

Review Preview to Chapter 10

The basic definitions and relations between angles covered in this exercise are used in the work of chapter 10.

Chapter 10 Numerical Applications of Geometry

General Comments

This chapter should be covered in 17-20 periods. The topic of constructing regular polyhedron can be expanded into a 3+ period craft session if so desired.

The material has been kept consistent with the title. Where a geometric idea has numerical applications it has been stated without proof and the applications investigated.

10.1 ANGLES
10.2 ANGLES OF A POLYGON

A Objectives: Define an angle as an amount of rotation.
Measure angles in degrees using a protractor.
Define seconds as a measure of an angle.
Convert from rotations to degrees and decimals of a degree.
Determine the number of sides of the common polygons from their names.
Divide polygons into triangles to determine the total degrees and hence the size of each angle of a regular polygon.

B How many factors has 360?
(24 including 360 and 1)
Why do you think 1 degree was defined $\frac{1}{360}$ of a rotation?

1 x 360
2 x 360
3 x 360
.
.
.

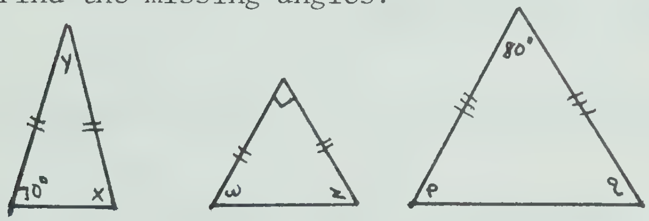
C Section 10.1 reviews basic ideas involving angles and rotation. Section 10.2 starts by giving a visual proof that a triangle has 180 degrees total in its angles and develops a method of determining the total number of degrees in any polygon, and from that the size of each angle of a regular polygon. The emphasis is on the process since it is unlikely that the student will remember a large number of polygons.

D Exercise 10.1 question 1 - the answers are to be left in degrees and decimal fractions of a degree. Although minutes and seconds of a degree will continue to be used in navigation, their use in other areas is likely to decline. Exercise 10.2 question 1 is an integral part of the process described above in C. Question 2 is a drawing exercise for those first finished.

10.3 CONGRUENCE

A Objectives: Recognize seemingly congruent figures by appearance.
Recognize congruent triangles from the minimum required information - SSS, SAS, ASA.
Watch congruent triangles to obtain data.

B Find the missing angles:



(x = 70°)
(y = 40°)
(w = z = 45°)
(p = q = 50°)

C The ideas of congruency have been developed intuitively since early elementary years. If your students have missed the experimental development of the triangle congruency relations it would be worth taking the time to do so now. One method is to have the students construct accurately triangles given various sets of data, then compare the triangles in the room by superposition against a window. Given AB = 5 cm, BC = 8 cm,- most triangles distinct. Given AB = 5 cm, BC = 8 cm, AC - 10 cm - all triangles congruent etc.

D Assuming that this topic is not new, the exercise has tried to involve the students in visually isolating a triangle when it is part of a more complicated figure.

10.4 PARALLEL LINES

A Objectives; Recognize seemingly parallel lines by appearance.
Identify parallel lines given equal corresponding or alternate angles or supplementary interior angles etc.
Locate equal angles given parallel lines.

B Solve the following equations:

a) $36^\circ = x - 180^\circ$ (144°) b) $180^\circ - x = 53^\circ$ (127°)

c) $x - 112^\circ = 180^\circ$ (68°) d) $180^\circ - x = x$ (90°)

C Investigation 10-4 will review the relations. Be sure the students construct large enough diagrams to allow them to use the protractors properly.

D Questions 7 to 11 help the students discover other geometric facts about the angles of a triangle and methods of constructing parallel lines. They will require some extra thought and equipment. These are good exercises for students working ahead of the class.

10.5 THE RIGHT ANGLED TRIANGLE AND PYTHAGORAS

A Objectives; Use the pythagorean theorem to test a triangle for the right angle.

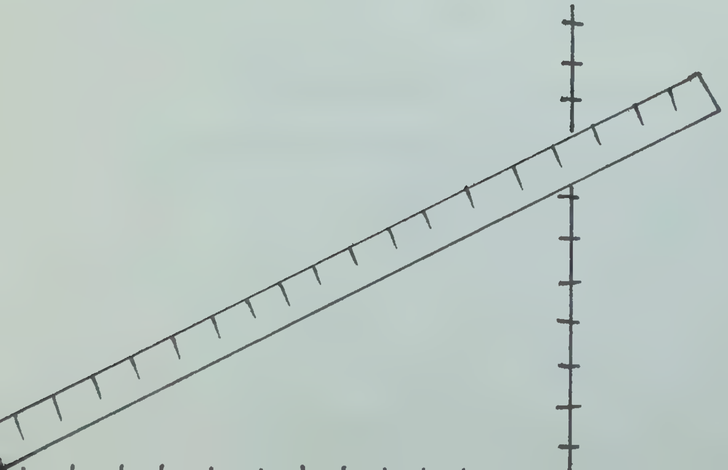
Use the pythagorean theorem to solve for one missing side of a right angled triangle.

Use the 3:4:5 and 5:12:13 triangles and their multiples to save calculation.

B Solve the following equations, $x > 0$

a) $x^2 + 16 = 25$ $(x = 3)$ b) $12^2 + x^2 = 169$ $(x = 5)$ c) $x^2 + 1600 = 41^2$ $(x = 9)$

C In investigation 10-5 question 4 may cause some problems. The construction is most conveniently done on graph paper with a strip of the graph paper used as the hypotenuse.



9^2 12^2	12^2 16^2	15^2 20^2
15^2 5^2	20^2 12^2	25^2 13^2
8^2	15^2	17^2
7^2	24^2	25^2
9	40	41
11	60	61
13	84	85

D Question 11 lays the foundation for later work in trigonometry. In question 12 the students may need help visualizing the triangles. A cut away cardboard box is of great assistance.

$AF^2 = 144 + 9$

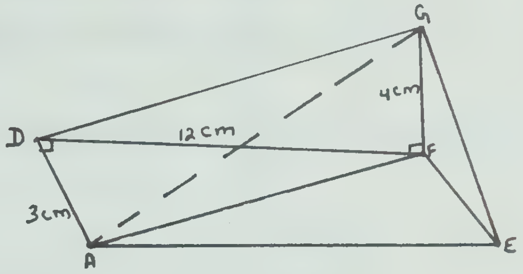
$= 153$

$AG^2 = 153 + 16$

$= 169$

$AG = 13$

See also question 14.



Question 13 - The flow is proportional to the area of the cross section of pipe which is proportional to the squares of the diameters.

10.6 PERIMETER, AREA AND VOLUME FORMULAS

A Objectives: Come to an intuitive understanding of the concepts of perimeter area and volume as measures of edge length and amounts of surface and space respectively.

Select the correct formula on the basis of the figure and the desired attribute and use it correctly.

Memorize the more common formulae.

B Draw as many different kinds of quadrilateral as you can name. (quadrilateral, trapezoid, kite, parallelogram, rhombus, rectangle, square).

C All the necessary information to do the exercise is provided on page 214. It is suggested that this exercise be used to have the students work independently on an 'individual help as required' basis.

D Questions 11 to 16 require adding or subtracting of areas or volumes to get the required answer. A possible project is to make a bristol board model of the shape or solid and show the calculation on the figure.

10.7 PRISMS, PYRAMIDS AND CONES

A Objectives: Recognize a prism, pyramid or cone and be aware of their properties.

Select and use the correct formula for the volume of a pyramid, prism or cone.

Calculate surface areas using properties of plane figures.

B Start right into investigations.

C If the students are to complete the construction portions of the questions they will require construction paper, scissors and glue. Note in part 4 V + F = 2 for all polyhedrons. A cone is not a polyhedron.

10.8 CONSTRUCTING THE REGULAR POLYHEDRONS

A Objectives: Recognize and name the regular polyhedrons.
Appreciate the symmetries of the figures.
Test the V + F - E = 2 relation.

B Start right into the investigation.

C This is a good opportunity for students to work in pairs. For some of these constructions an extra pair of hands at critical moments is essential. Pipe cleaners may be used to join straws, as shown in the text with paper clips. Tooth picks joined by glue or small balls of plasticine or putty is another alternative.

Review Preview to Chapter 11

This exercise reviews work with proportion and triangles which will be required by the students for trigonometry in chapter 11.

Chapter 11 TRIGONOMETRY

General Comments

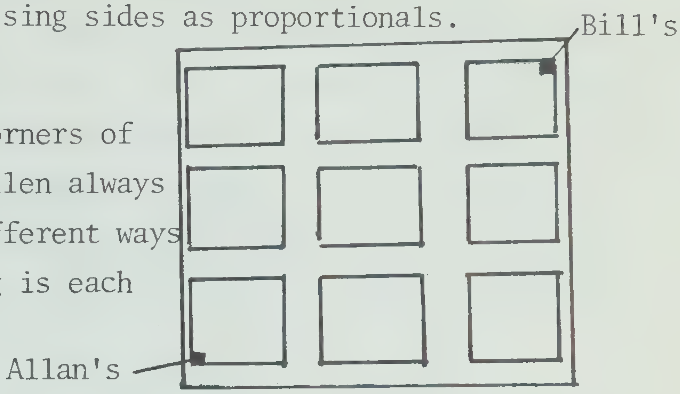
This chapter should be covered in 13-15 periods. The material lends itself to a practical approach leading to an outside surveying exercise and a shop problems exercise. Start early looking for a suitable location and gathering measuring supplies if you intend to do section 11.5 investigations. The machine shop teacher will be able to verify the trigonometry skills required by his students if your school is so equipped.

11.1 SIMILAR TRIANGLES

A Objectives; Recognize similar figures intuitively, Given two similar right angled triangles be able to set up proportions between the sides.

Calculate the length of missing sides as proportionals.

B Allan and Bill live at opposite corners of a square nine city blocks. If Allen always travels north or east how many different ways can he travel to Bill's? How long is each trip? (20 ways, 6 blocks)



C This section is strictly a preliminary to the introduction of trigonometric ratios. It is not intended to be an in depth study of similar triangles. From the investigation the students should get the idea that the corresponding sides of similar triangles are proportional. The idea of corresponding sides being the sides opposite the equal angles may require extra emphasis.

D The problem of corresponding sides is highlighted in question 3.

11.2 THE TANGENT RATIO

A Objectives; Give the definition of tangent, pertaining to an angle in a right angled triangle.

Given a right angled triangle with known sides find the tangents of the acute angles.

Use the tangent table to solve for the sides about the right angle.

Use tangent tables to find acute angles on right triangles.

B Solve to 3 figures;

a) $\frac{y}{5} = 2.53$ b) $\frac{y}{4.2} = 0.95$ c) $\frac{4}{x} = 3.4$ d) $\frac{7.8}{x} = 0.21$

(12.7, 3.99, 1.18, 37.1)

C Much oral practice needs to be done on drilling opposite and adjacent sides and on forming the tangent ratios from the sides.

D Finding the side opposite the given angle seems to follow more easily for the students. Question 3 limits itself to the type of solutions. Question 4 deals with solving for the adjacent side and might well be left for a second day.

11.3 THE SINE AND COSINE RATIOS

A Objectives: Give the definition of the sine and cosine pertaining to an angle in a right angled triangle.

Given a right angled triangle with known sides find the sine and cosine of the acute angles.

Use the sine and cosine tables to solve for the sides about the right angle of a right angled triangle.

B Find the missing angle in each of the following triangles;

∠A	90°	45°	52°		
∠B	30°	90°		50°	90°
∠C			90°	40°	75°

(60°) (45°) (38°) (90°) (15°)

C This section expands the topic to cover sine and cosine. It does not get into solving for angles given the sine or cosine nor does it solve for the hypotenuse. This would be a good time to have the students compare the amount of work required to solve for a missing side using trigonometry with using the pythagorean theorem.

D The exercise reviews the work on tangents as well as exercising sine and cosine.

11.4 RIGHT ANGLES

A Objectives: Define the primary trigonometric ratios in terms of adjacent, opposite and hypotenuse or for right angled triangle ABC in terms of a,b,c.

Recognize what is meant by solving a triangle.

Given a second angle and a side, or two sides of a right angled triangle, solve the triangle.

B Use tables to find the following values:

a) sin 35°, cos 55° b) sin 54°, cos 26° c) sin 45°, cos 45°
d) sin 77°, cos 13° e) sin 8°, cos 82° f) sin 90°, cos 0°

What do you notice about the sum of the angles? (90)

What do you call angles that sum 90°? (complementary)

Notice that the cosine is the sine of the complement hence co-sine.

Notice that the sine and cosine tables are really the same table written in opposite order.

C The only new ideas here are the definition of side 'a' being opposite angle A etc. and the definition of a solved triangle. The rest is more practice of the material from sections 11.2 and 11.3.

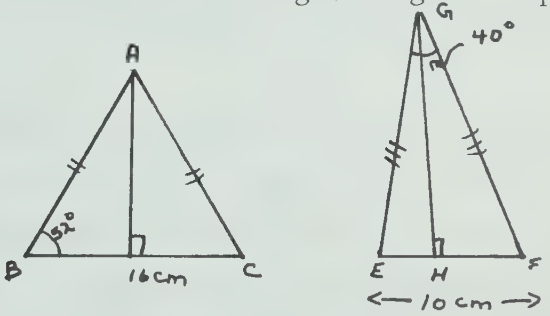
D The choice of ratio may cause some difficulty from question 7 on. The equation $\frac{\text{unknown}}{\text{known}}$ = (ratio?) where known is the known side and unknown is the side we wish to find, and Ø is a known angle will help the student organize the information

11.5 APPLICATIONS OF TRIGONOMETRY

11.6 SHOP PROBLEMS

A Objectives: Apply the skill of solving triangles to practical problems.

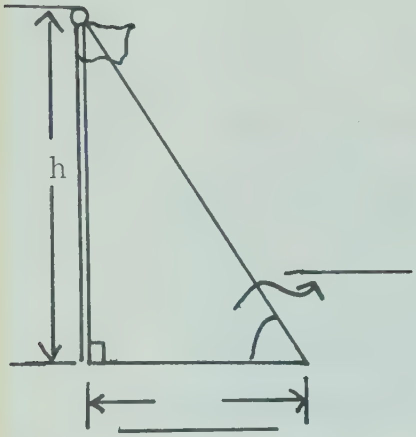
B (Applies to 11.6)

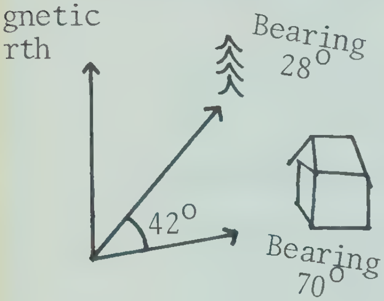


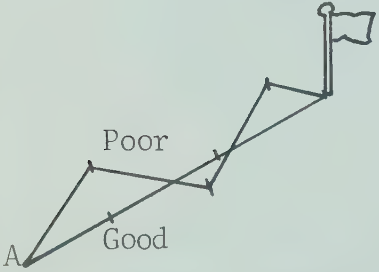
The isosceles triangles above have been divided into right angled triangles. Add as much information to the diagram as possible.
($\angle C = 42^\circ$, $\angle BAD = \angle DAC = 38^\circ$, BD = DC = 8cm, $\angle E = \angle F = 70^\circ$, EH = HF = 5cm)

C An outside measuring exercise at this point is valuable if well prepared.
Some suggestions: 3 to 5 students per team - 1 recorder group leader
- 2 measurers or 2 angle measurers and 2 distance measurers.

-prepare a duplicated sheet that has a diagram and shows the distance and angles to be measured with spaces for data.







-don't have more than 3 groups doing the same project in the same time slot.

-once the data is collected return to the classroom and each student gets a clean data sheet to copy the data onto and complete the calculations.

-students in the same group should get the same answers from the same data.

-average the answers of different groups doing the same problem to get a class answer.

-where possible have students get a checking answer by direct measurement after the class answer is arrived at.

Example - tie the end of the measuring tape to the flag halyard and run it up the pole.

-horizontal angles can be measured using an orienteering type compass.

-when taping distances beyond 1 tape length the rear taper keeps the front taper on line with hand signals. The front taper keeps a supply of sticks and pushes one into the ground to mark the end of the tape length. The rear taper collects the sticks and this gives a count of the number of tape lengths.

D Exercise 11-6 is designed for students in the technical shop programme. However, any student can profit from overcoming the challenge of a problem for students who cannot visualize these diagrams as physical objects, a practical assignment might be substituted. A good example of a wedge is a cedar shingle. Tree felling wedges (not log splitting) can be purchased inexpensively. These might be more meaningful for some students to work with.

Section 11.7 is optional and could be used as an alternative for students for whom this work is deemed unsuitable.

11.7 RECIPROCAL TRIGONOMETRIC RATIOS (optional)

A Objectives; Define the reciprocal trigonometric ratios.

Select the ratio, primary or reciprocal that simplifies the calculation.

B Solve:

$\frac{x}{4} = 2.4$ $\frac{x}{12.5} = 0.93$ $\frac{x}{53} = 0.65$

(9.6) (11.625) (34.45)

C If the equation $\frac{\text{unknown}}{\text{known}} = (\text{ratio?})$ is used to decide which trig. ratio to use in solving for a side the solution always involves multiplication. At this level of mathematics that is probably the most meaningful selling point for learning to use the reciprocal ratios. With the use of calculators it is a marginal gain.

D The exercise has been kept short and simple. This is an introduction to the topic only.

Review Preview to Chapter 12

This section reviews the basic constructions and allows the student to compare various methods. Again the emphasis should be on practical skill development rather than theory.

Chapter 12 GEOMETRIC VECTORS

General Comments

This chapter should be covered in 8 - 10 periods. This chapter deals with the addition of vectors and the solution of vector problems entirely through scale diagrams. The application of trigonometry at this level has not been attempted. The last section on navigation problems is optional.
Graph paper can be used to advantage throughout this chapter.

12.1 DIRECTED LINE SEGMENTS

12.2 COLLINEAR VECTORS

A Objectives; Distinguish between scalar and vector quantities.
Represent vector quantities by scale diagram.
Recognize collinear vectors.
Find the resultant of a system of collinear vectors.

B Simplify

(a) 6 + 2 + (-4), (4)
(b) 3 + (-7) + 2, (-2)
(c) 5 + 8 + (-12), (1)
(d) -6 + 9 + (-5), (-2)
(e) 4 + 6 + 1, (11)
(f) -3 + (-5) + (-11), (-19)

C An early emphasis on neat, accurate, large diagrams will pay dividends when section 12.3 involving the addition of non-collinear vectors is covered.

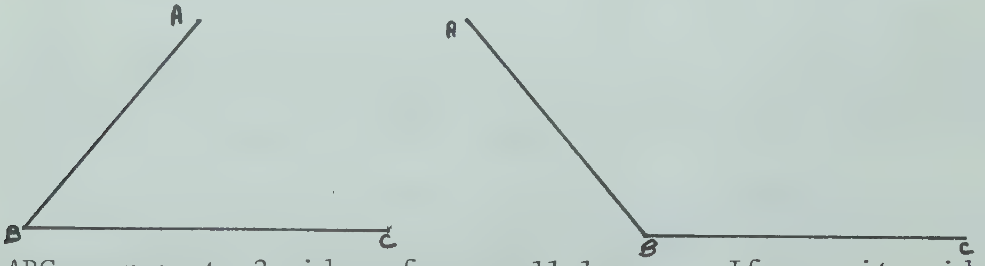
Few students have any concept of the newton force. If spring scales calibrated in newtons are available it would be helpful to bring them in and have the students get a feel for the forces involved. Alternately their mass in kilograms x 9.8 gives the gravitational force in newtons acting on their bodies.

D These early exercises are most easily done on graph paper. Exercise 12.2 question 4 touches briefly on scalar multiplication without naming it as such. This would be a good time to have the students recall the work with matrices.

12.3 ADDITION OF VECTORS (NON-COLLINEAR)

A Objectives: Solve problems involving the addition of non-collinear vectors using scale diagrams and either the head to tail method or the parallelogram method.

B

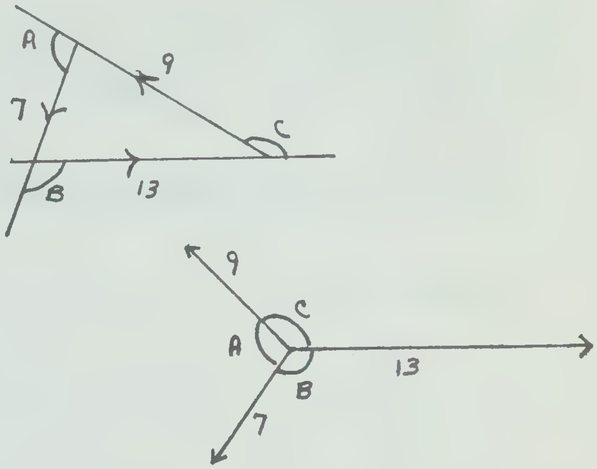


ABC represents 2 sides of a parallelogram. If opposite sides of a parallelogram are equal,complete the construction using compasses.

C Both the head to tail and parallelogram method of adding vectors are discussed. It is suggested that both be taken up but that the students be allowed to choose when doing questions. In questions involving bearings,graph paper is helpful in keeping the direcions in line. Where more than two vectors are involved the head to tail or polygon method should be encouraged.

D Question 8 involves finding the angles between forces in equilibrium. Note that the angles between the forces are not those found in the triangle but their supplements.

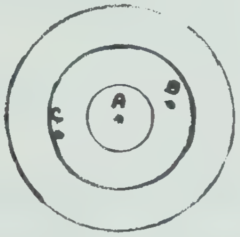
Question 9 leads to section 12.4. In it the current of a stream acts on a boat where no allowance has been made. Section 12.4 deals with methods of predicting and compensating for drift.

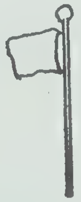


12.4 NAVIGATION PROBLEMS (optional)

A Objectives: Lay out a course to allow for a cross-wind or water current.

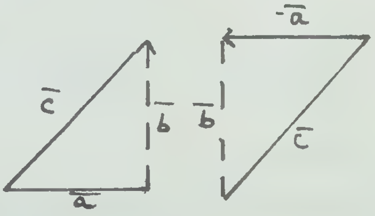
B





On a rifle range a strong wind is blowing in the direction indicated by the flag. At extreme range the bullet will be deflected. Will it hit at B or C if the aiming point is A? (C).
To hit A where should the shooter aim? (B)

C Reduced to algebra this section really says, 'If $\bar{a} + \bar{b} = \bar{c}$, find $\bar{b}$ given $\bar{a}$ and $\bar{c}$'. Which reduces to $\bar{b} = \bar{c} - \bar{a}$ or $\bar{b} = \bar{c} + (-\bar{a})$



If your students have been using head to tail method of adding vectors this approach might be suitable. Note that a west wind blows from the west etc.

D 105 km = 105 nautical miles per hour.
Note question 7 gives the heading of the aircraft and asks for the direction of travel, all others ask for the heading to achieve a desired direction of travel.

Review Preview to Chapter 13

This exercise reviews the work of graphing points and equations. The skills will be required in chapter 13.

Chapter 13 Co-Ordinate Geometry

General Comments

This chapter should be covered in 12-15 periods. It builds on the work of Relations in Chapter 5 formalizing the idea of slope and investigating parallel and perpendicular lines and distance between points.

13.1 SLOPE

13.2 SLOPE OF A LINE

A Objectives: Define slope.
Determine the slope of a line segment between two given points.
Interpret positive and negative slopes in terms of general direction.
Interpret zero slope and undefined slope.
Find the slope of a line given its equation.
Graph a line given a point and slope.

B Simplify
(a) 4 - 6, (-2) (b) - 2 - 5, (-7) (c) 7 - (-3), (10)
(d) - 8 - (-4), (-4) (e) 9 - (-12), (21) (f) 4.7 - 8.3, (-3.6)

C In chapter 5 the students were shown that in the form $y = mx + b$, equations with the same 'm' value represent parallel lines. Here the idea of slope is formalized.

Investigation 13-1 is establishing the equivalence of $\frac{y2 - y1}{x2 - x1}$ for two points on the line and 'm' in the equation form $y = mx + b$.
Section 13.2 example 3 establishes a quick way of graphing a line given the slope and a point without first finding the co-ordinates of a second point.

D Exercise 13-1 question 3 and 4 investigate the idea of collinear points. Exercise 13-2 questions 2 and 4 deal with the problem of zero and undefined slope in $y = mx + b$. this is the place to come back to the idea that the equation of a line makes a statement that the co-ordinates of every point on the line must make true. If the line is vertical every x co-ordinate must have a given value. If the line is horizontal the y co-ordinate is constant.

13.3 PARALLEL LINES

13.4 PERPENDICULAR LINES

A Objectives: Determine if line segments are parallel or perpendicular given two pair of points.

Determine if lines are parallel or perpendicular given their equations.

B Do investigation 13-2 to begin section 13.3
to begin section 13-4 find the product;
(a) $\frac{3}{4} \times \frac{8}{5}$, $\frac{6}{5}$ (b) $\frac{2}{3} \times \frac{-3}{2}$, (-1), (c) $\frac{-5}{7} \times \frac{14}{14}$, $(\frac{-2}{3})$

(d) $\frac{-4}{9} \times \frac{-18}{-8}$, (-1)

C Section 13.3 example 2 and question 8 may be kept as a challenge for your more able students.

D Exercise 13-3 question 4 deals with the problem of equating undefined slopes. If the values are undefined we may not say that they are equal, but the lines are parallel. Similarly in exercise 13-4 question 2 we may not equate 0 x undefined with -1 but the lines are perpendicular. In Exercise 13-4 question 5 an accurate sketch will determine which of the three line segments defined by each set of points should be tested.

13.4 LENGTH OF A LINE SEGMENT

A Objectives: Find the length of a line segment given the co-ordinates of its end points.

B Use the table on page 47 to find a decimal value rounded off to 3 figures:

a) $\sqrt{35}$, (5.92) b) $\sqrt{17}$, (4.12) c) $\sqrt{21}$, (4.58)
d) $\sqrt{148}$, ($2\sqrt{37} = 12.2$)

C For most of this work the answers may be left in radical form or used as an opportunity to practice the estimation of square roots at the teacher's discretion. Encourage the students to save time and effort by spotting line segments parallel to the axis where the distance is a simple run or rise.

D Question 5 and 6 deal with mid-points of line segments.
The formula $\left(\frac{X_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ for mid points is so simple and direct it would be a shame not to share it with your students as an optional extra thrown in at no additional charge.

13.6 APPLICATIONS

A Objectives: Use graphs to compare data and derive information from relations.
Be introduced to the idea of slope representing a rate of change such as km/h or $/km

B Find the following average rates;
a) 2 h to travel 100 km, (50 km/h)
b) 5 tapes cost $20, ($4/tape)
c) 150 cm^3 have mass 750 g, (5 g/cm^3)
d) 80 L of gas cost $24, (30¢/L)

C In chapter 5 the students used graphs to interpolate and extrapolate values for the dependent variable. In this section we are taking them one step further to gain information not only from co-ordinates of points on the graph but also from the slope of the graph and from the intercept. Some students may recall the work on partial variation, in which case point out that in Chapter 7 they were given a mathematical statement of the relation, now they are able to derive their own from sets of values (under assumption that it is linear).

D Questions 3 - 7 are suitable for small groups to use in the preparation of display solutions on large graph sheets. This process tends to help students who are having difficulty with the topic to consolidate the ideas.

Pretest for Review Preview

Exercise 1 topics

1. Add

a) $\begin{array}{r} 4.62 \\ 59.71 \\ \underline{84.23} \end{array}$ b) $\begin{array}{r} 4.357 \\ 2.643 \\ \underline{0.821} \end{array}$ c) $\begin{array}{r} 344.2 \\ 165.8 \\ \underline{384.7} \end{array}$

2. Subtract

a) $\begin{array}{r} 4.359 \\ \underline{3.846} \end{array}$ b) $\begin{array}{r} 784.3 \\ 92.8 \end{array}$ c) $\begin{array}{r} 66.84 \\ \underline{38.56} \end{array}$

3. Simplify

a) 10.65 - 8.214

b) 3.745 + 29.6 + 0.403

c) 1 - 0.764

Exercise 2 topics

4a) 10 x 3.89 b) 0.1 x 23.7 c) 0.001 x 8.47

5. Multiply

a) $\begin{array}{r} 13.3 \\ \underline{\text{x} \quad 5} \end{array}$ b) $\begin{array}{r} 8.46 \\ \underline{\text{x } 2.7} \end{array}$ c) $\begin{array}{r} 0.215 \\ \underline{\text{x } 0.93} \end{array}$

6. Divide

a) $\frac{71.4}{21}$ b) $\frac{0.4674}{0.82}$ c) $\frac{0.02408}{5.6}$

Exercise 4 topics

7a) 10% of \$47.50 b) 25% of \$29.00 c) 150% of \$84.50

8. Express the first number as a per cent of these conditions:

a) \$47, \$94 c) \$24.50, \$98 c) \$75, \$50

Notes and answers on pretest

If a student gets all parts of a question correct then they might be exempted from that topic in the R. P. Exercise.

1a)	148.56	b)	7.821	c)	894.7
2a)	0.513	b)	691.5	c)	28.28
3a)	2.436	b)	33.748	c)	0.236
4a)	38.9	b)	2.37	c)	0.00847
5a)	66.5	b)	22.842	c)	0.19995
6a)	3.4	b)	0.57	c)	0.0043
7a)	$4.75	b)	$7.25	c)	$126.75
8a)	50%	b)	25%	c)	150%

Test Chapter 1. Number applications

1. Complete the following calculations. Round off the answer to the nearest $\frac{1}{100}$.

   a) $\begin{array}{r} 642.93 \\ +859.36 \\ \hline \end{array}$ b) $\begin{array}{r} 47.38 \\ -29.23 \\ \hline \end{array}$ c) $\begin{array}{r} 54.3 \\ \times 96.8 \\ \hline \end{array}$ d) $\frac{47.2}{9.81}$

2. Mary Ellen works in a bakery for $5.20/hr.

   a) How much does she make in a 40 hr. week?

   b) What will she earn over 8 weeks this summer?

3. Simplify the following expressions:

   a) $5 \times 8 - 4$

   b) $7 + 2 \times 3$

   c) $12 - (4 + 5)$

   d) $\frac{12 + 33}{19 - 14}$

   e) $3\ (4 + 6^2)$

   f) $(14 - 6) + (17 - 4)$

   g) $\frac{4.63 \times 5.82}{3.75} + \frac{24.9^2}{42.3 - 26.2}$

4a) Round off the numbers to the nearest 10, do the calculation and round off your answer to the nearest 1000.

694×387

4b) Round off the numbers to the nearest $\frac{1}{100}$

$27.68 \div 16.73$

5. Write in shorter form. (do not work out)

a) $3 \times 3 \times 3 \times 3 - 6 \times 6 \times 6$

b) $7 \times 7 \times 7 \times 7 \times 7 \times 7 + \dfrac{1}{2 \times 2 \times 2}$

6. Evaluate

a) 5^2

b) 4^3

c) 2^{-4}

d) 12^0

e) $2^3 + 3^2$

f) $7^3 - 9^2$

g) $3.6^4 \times 4.9^5$ (nearest whole number)

7a) Classify as approximate or exact numbers by writing an 'A' or an 'E' after each

i) A box 7.6 cm long

ii) 16.4 L/100km gas consumption

iii) 96 freight cars in a train

iv) 0.65 kg of bacon

7b) Complete the following table

Decimal Number	Standard Notation	Number of Significant Digits
4265		
47.5		
0.7203		
0.006		
	2.97×10^4	
	4.5×10^{-1}	
	6.04×10^{-5}	
	7.9638×10^3	

8. Without using calculators, estimate the answers to the question to be put on the board.

a) (see notes)

Test Chapter 1 Notes and Answers

The test assumes the use of calculators, otherwise omit questions 3g) and 6g).

For question 8 have calculators put away and put on the board:

a) 598 x 61.3 b) 4890 ÷ 8.4

Answers

1. 1502.29 b) 18.15 c) 5256.24 d) 4.81

2a) $208 b) $1664

3a) 36 b) 13 c) 3 d) 9

e) 120 f) 21 g) 45.695....

4a) 690 x 390 = 269000 b) 27.7 ÷ 16.7 = 1.66

5a) $3^4 - 6^3$ b) $7^6 + 2^{-3}$ or $7^6 + \frac{1}{2^3}$

6a) 25 b) 64 c) $\frac{1}{16}$ d) 1

e) 17 f) 262 g) 474 450

7a) i) A ii) A iii) E iv) A

b)

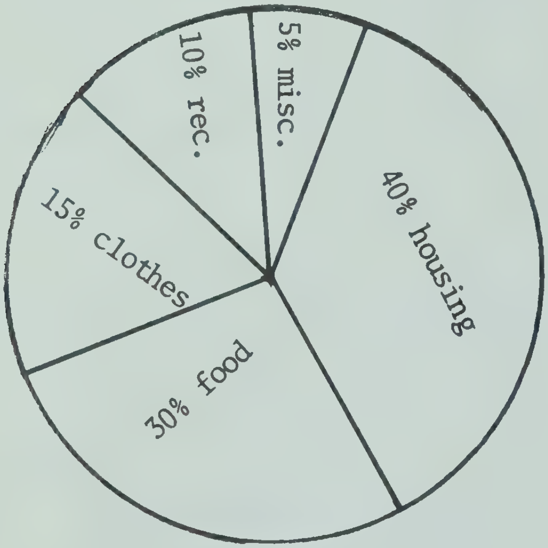
4265	4.265×10^3	4
47.5	4.75×10	3
0.7203	7.203×10^{-1}	4
0.006	6×10^{-3}	1
29 700	2.97×10^4	3
0.45	4.5×10^{-1}	2
0.000 060 4	6.04×10^{-5}	3
7963.8	$7.963\,8 \times 10^3$	5

8a) 36 000 ± 15 000 b) 600 ± 300

Test Chapter 2 Statistics

1. If you were drawing graphs to illustrate the following sets of data, what type of graph would you choose?
   a) Number of students participating in football, basketball, soccer, baseball, track and field.
   b) How a student divides his school day among travel, classes, study, meals, recreation, sleep.
   c) The daily height of a bean plant in cm. being grown in a science experiment.

2.



The circle graph shows how the Jacobs spend their net income. If their take home pay is $850 per month, how much do they spend monthly on housing?

clothes?

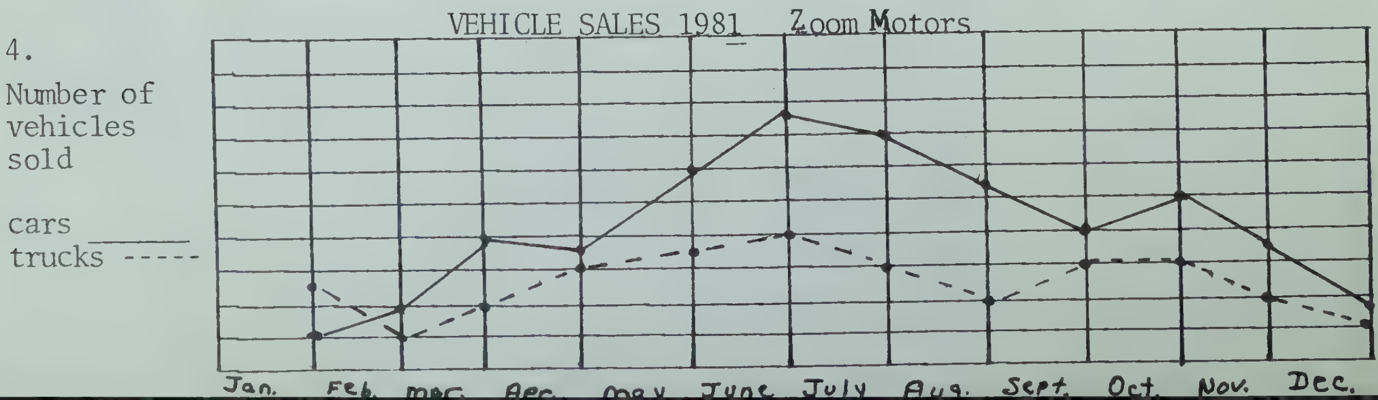
How much per year do they spend on food?

recreation?

3. Spitball McQueen had the following earned run record before he was traded: 3, 2, 0, 4, 3, 6, 5, 9.

What was his earned run average?

4.



4. Answer from the graph:

   a) How many cars were sold in May?

   b) How many trucks were sold in October?

   c) Which month had the highest vehicle sales?

   How many were sold?

   d) Which month had the lowest vehicle sales?

   How many were sold?

   e) How many trucks were sold from May to August inclusive?

   f) What was the mean number of cars sold per month?

   Draw a line across the graph at the mean number.

5. The following set of data represents marks on a test out of 25:

20 17 15 10 16 14 18 16 15 19 21

19 16 14 15 17 18 16 12 18 20 23

   a) Find the median score.

   b) Complete the frequency distribution table using an interval of 2.

Interval	Tally	Frequency
10 - 11		

   c) Construct a histogram with the data.

   d) If Jan Shorten got a mark of 18, what was her percentile rank?

Test Chapter 2 Notes and Answers

This test should be supplemented by a project mark.

Answers

1a) Bar graph b) Circle graph c) line graph

2. housing $340/month, clothes $127.50/month

   food $3060/a, recreation $1020/a

3. 4.0 E.R.A.

4a) 12 b) 6 c) June 25 d) Dec. 3

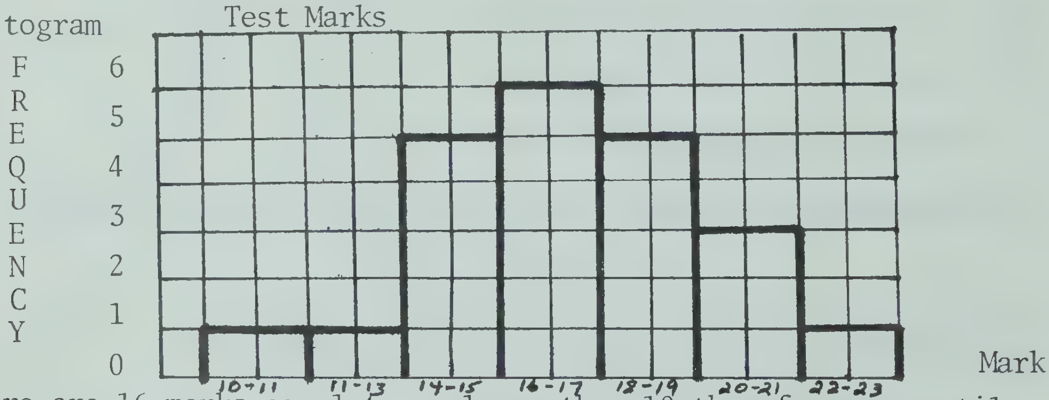
e) 30 f) $\frac{101}{12} = 8.4$ cars/month

5a) median $= \frac{16 + 17}{2} = 16.5$

b)

Interval	Tally	Frequency
10 - 11	1	1
12 - 13	1	1
14 - 15	~~1111~~	5
16 - 17	~~1111~~ 1	6
18 - 19	~~1111~~	5
20 - 21	111	3
22 - 23	1	1

c) Histogram



d) There are 16 marks equal to or lower than 18 therefore percentile

$= \frac{16}{22} \times 100 = 72.7$ percentile (accept 73 percentile)

Chapter 3 Test Part A (after sec. 313)

1. Write as a common fraction in its simplest terms:

a) 0.5 b) 0.75 c) 0.84 d) 5.7 e) $0.\dot{5}\dot{1}$

2. Write in decimal form:

a) $\frac{1}{4}$ b) $\frac{3}{5}$ c) $1\frac{3}{8}$ d) $\frac{47}{50}$ e) $\frac{2}{3}$

3. Classify as rational (Q) or irrational ($\overline{Q}$):

a) $\frac{5}{9}$ b) $\sqrt{7}$ c) 0.63 d) 0.717717771... e) $\sqrt{36}$

4. By factoring determine which of the following are rational expressions and evaluate them:

a) $\sqrt{180}$ b) $\sqrt{576}$ c) $\sqrt{441}$

5. Write as mixed radicals

a) $\sqrt{8}$ b) $\sqrt{27}$ c) $\sqrt{72}$

6. Write as an entire radical:

a) $2\sqrt{3}$ b) $5\sqrt{2}$ c) $4\sqrt{5}$

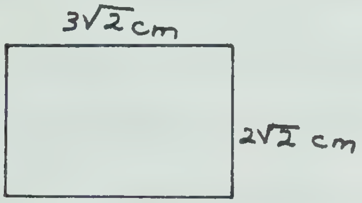
7. Multiply and leave the answer in its simplest form:

a) $\sqrt{3} \times \sqrt{3}$ b) $\sqrt{5} \times \sqrt{2}$ c) $2\sqrt{3} \times 4\sqrt{2}$

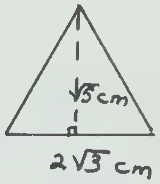
d) $2\sqrt{6} \times 3\sqrt{2}$ e) $4\sqrt{5} \times \sqrt{10}$ f) $3\sqrt{6} \times 2\sqrt{21}$

8. Find the areas of the following figures:

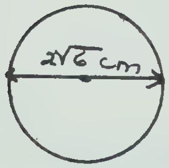
a)



b)



c)



Chapter 3 Test Part B (after Chapter completed)

1. Write as a mixed radical in its simplest form:

a) $\sqrt{12}$ b) $\sqrt{32}$ c) $3\sqrt{75}$

2. Simplify:

a) $\sqrt{3} + 2\sqrt{3}$ b) $3\sqrt{5} + 6\sqrt{3} - 2\sqrt{5} + \sqrt{3}$

c) $2\sqrt{12} + 4\sqrt{3}$ d) $5\sqrt{8} + 2\sqrt{18} - 4\sqrt{32}$

3. Select the best estimate for the following expressions:

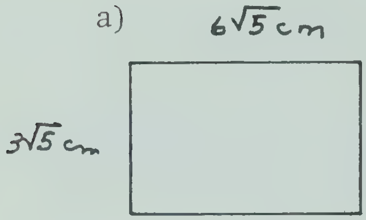
a) $\sqrt{3} \doteq$ (2.0, 1.7, 1.1) b) $2\sqrt{10} \doteq$ (6.3, 7.0, 7.5)

c) $\sqrt{50} + \sqrt{98} \doteq$ (12, 15, 17) d) $\sqrt{500} \doteq$ (22, 25, 27)

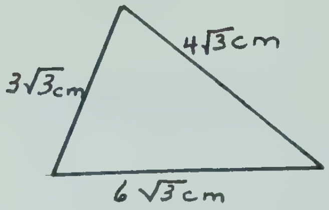
e) $\sqrt{0.007} \doteq$ (0.8, 0.08, 0.008) f) $\sqrt{1\,460\,000} \doteq$ (12, 120, 1200)

4. Estimate the perimeter or circumference of the following figures to one place of decimals.

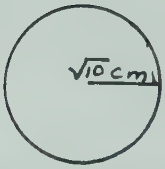
a)



b)



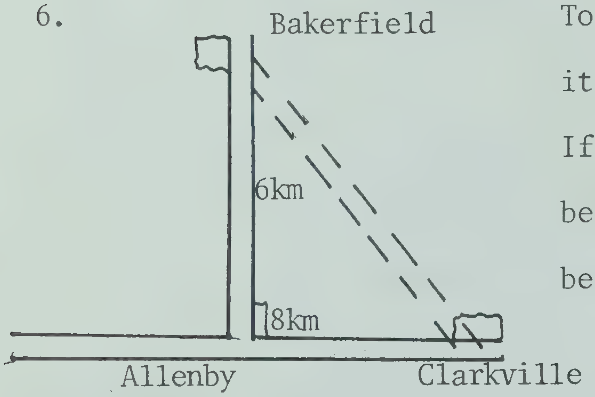
c)



5. The time for an object to fall is given by $t = \sqrt{\dfrac{h}{4.9}}$ (t = time in seconds) (h = height in m)

Find the time for a ball to fall from a tower 490 m tall.

6.



To travel from Bakersfield to Clarksville it is necessary to go by way of Allenby. If a new highway was built directly between the towers how long would it be? How much distance would be saved?

Test Chapter 3 Notes and Answers

This test is in two parts so that the chapter may be split into a section on fractions and roots as radicals and a section on decimal approximations.

Answers Part A

1a) $\frac{1}{2}$ b) $\frac{3}{4}$ c) $\frac{21}{25}$ d) $5\frac{7}{10}$ or $\frac{57}{10}$ e) $\frac{17}{33}$

2a) 0.25 b) 0.6 c) 1.375 d) 0.94 e) $0.\dot{6}$

3a) Q b) $\overline{Q}$ c) Q d) $\overline{Q}$ e) Q

4a) $\overline{Q}$ b) 24 c) 21

5a) $2\sqrt{2}$ b) $3\sqrt{3}$ c) $6\sqrt{2}$

6a) $\sqrt{12}$ b) $\sqrt{50}$ c) $\sqrt{80}$

7a) 3 b) $\sqrt{10}$ c) $8\sqrt{6}$ d) $12\sqrt{3}$ e) $20\sqrt{2}$ f) $18\sqrt{14}$

8a) 12 cm^2 b) $\sqrt{15}$ cm^2 c) 18.84 cm^2

Part B

1a) $2\sqrt{3}$ b) $4\sqrt{2}$ c) $15\sqrt{3}$

2a) $3\sqrt{3}$ b) $\sqrt{5} + 7\sqrt{3}$ c) $8\sqrt{3}$ d) 0

3a) 1.7 b) 6.3 c) 17 d) 22 e) 0.08 f) 1200

4a) 40.2 cm b) 22.5 cm c) 19.9 cm (all answers ± 0.5)

5. 10 s 6 10 km, save 4 km.

Chapter 4 Test Part A Powers and Polynomials

1. Simplify

a) $2a^3 \times 3a^4$

b) $6a^2b \times 3ab^4c$

c) $20a^4 \div 5a^3$

d) $36m^2n^3 \div 12m^2n^2$

e) $(-3a^2b^4)\ (5ab^3)$

f) $(-16m^4n^6) \div (-8m^2n^6)$

g) $(3x^3)^2$

h) $(2mn^2)^4$

i) $(\frac{16x^5}{8x^2})^3$

j) $\frac{(-3x^2)\ (12xy^3)}{-4xy}$

k) $(2y^3)\ (3y-^1)$

i) $(3a^4b^{-3})(-2a^{-2}b^{-1})$

m) $(15a^4b^{-3}) \div (5a^6b^2)$

n) $(4a^2b^{-4})^{-2}$

2. Evaluate

a) $2^2 \times 2^{-3}$

b) 6^0

c) $3^{-2} + 2^3$

d) (3/4)-1

3. If a = 1, b = -2 and c = 3 evaluate:

a) $2a + 3b - c$

b) $3abc^2$

c) $3a^2 + 4b^2 - 2c^2$

d) $2ab + 3bc + 4ac$

4. Simplify

a) $2(x + 4) + 3(x - 2)$

b) $3x(x - 5y) - 2y(x + 3y)$

c) $(x + 4)\ (x - 5)$

d) $(2x - 3y)(3x - y)$

e) $(x + 4)^2$

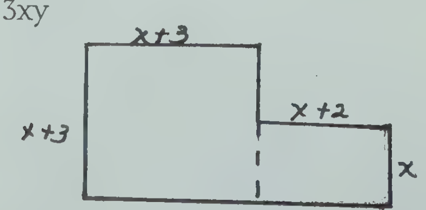
f) $(2a - 3b)^2$

g) $4(x - 2)\ (x + 5)$

h) $\frac{27a^2b^3}{-9ab^2}$

i) $\frac{6x^2y - 9xy^3}{3xy}$

5.



a) find an expression for the total area and simplify.

b) find the area if x = 2cm.

6. Divide $x^3 + 5x^2 - 4x - 2$ by $x - 1$

Chapter 4 Test Part B Factoring

1. Which of the following expressions are in factored form:

a) $4(x + 2)$ b) $(3x - 2)(x - 4) + 2$
c) $4x(x - 1) + 3x(x + 2)$ d) $3(x + 4)(x - 1)$

2. Factor the following common factor expressions:

a) $6x^3 - 3x^2$ b) $5a^2b^3 + 20ab^2$
c) $12m^3 + 6m^2$ d) $8m^3 + 6m^2 - 4m$
e) $x(x + 2) + 3(x + 2)$ f) $m(m - 3) - 4(m - 3)$

3. Factor the following differences of squares:

a) $x^2 - y^2$ b) $a^2 - 9$
c) $9m^2 - 4$ d) $16m^2 - 25n^2$

4. Factor the following trinomials:

a) $x^2 + 5x + 6$ b) $x^2 + 6x + 9$
c) $x^2 - 2x + 1$ d) $x^2 - 9x + 20$
e) $x^2 + 3x - 4$ f) $x^2 - x - 12$

5. Factor the following expressions:

a) $16x^2 - 9$ b) $8x^2 - 4x$
c) $3a(x - 2) + 6a^2$ d) $x^2 - 8x + 12$

Test Chapter 4 Notes and Asnwers

These tests have been kept to the basics with part A covering the first ten sections and part B covering the factoring so that the chapter may be split into two separate units.

Answers Part A

1 a) $6a^7$ b) $18a^3b^5c$ c) $4a$ d) $3n$ e) $-15a^3b^7$ f) $2m^2$
g) $9x^6$ h) $16m^4n^8$ 1) $8x^9$ j) $9x^2y^2$ k) $6y^2$ 1) $-6a^2b^{-4}$
m) $5a-^{2}b^{-5}$ n) $\frac{b^8}{16a^4}$

2 a) ½ b) 1 c) $8\frac{1}{9}$ d) $\frac{4}{3}$

3 a) -7 b) -54 c) 1 d) -10

4. a) $5x + 2$ b) $3x^2 - 17xy - 6y^2$ c) $x^2 - x - 20$
d) $6x^2 - 11xy + 3y^2$ e) $x^2 + 8x + 16$ f) $4a^2 - 12ab + 9b^2$
g) $4x^2 + 12x - 40$ h) $-3ab$ i) $2x - 3y^2$

5. a) $2x^2 + 8x + 9$ b) 33 cm^2

6. $x^2 + 6x + 2$

PART B

1 a) and d)
2 a) $3x^2(2x - 1)$ b) $5ab^2(ab + 4)$ c) $6m^2(2m + 1)$
d) $2m(4m^2 + 3m - 2)$ e) $(x + 2)(x + 3)$ f) $(m - 3)(m - 4)$

3, a) $(x + y)(x - y)$ b) $(a - 3)(a + 3)$ c) $(3m - 2)(3m + 2)$
d) $(4m - 5n)(4m + 5n)$
4. a) $(x + 2)(x + 3)$ b) $x + 3)^2$ c) $(x - 1)^2$
d) $(x - 4)(x - 5)$ e) $(x + 4)(x - 1)$ f) $(x - 4)(x + 3)$

5. a) $(4x - 3)(4x + 3)$ b) $4x(2x - 1)$
c) $3a(x - 2 + 2a)$ d) $(x - 6)(x - 2)$

Chapter 5 Test

1a) Complete the table for the relation $y = 3x - 2$

x	y
1	
2	
3	
4	

b) give the domain of the relation

c) give the range of the relation.

d) draw the graph.

2a) On a single set of axes graph the following relations where $x, y \in R$.
Determine at least 3 ordered pairs for each relation.
$y = 5x, \quad y = 5x + 3, \quad y = 5x - 2$

b) What do the three graphs have in common?

3a) Write the equation $4x + 3y = 12$ in the form $y = mx + b$.

b) Write the equation for a relation whose graph will have the same direction as the relation in a).

c) Write the equation for a relation whose graph will have the same y-intercept as the relation in a).

4a) Graph the relation $y = 4x + 3$, $x,y \in R$.

b) Mark the areas on the graph where $y < 4x + 3$
and $y > 4x + 3$

5. Graph the ordered pairs given in the following table of values, join the points with a smooth curve and extend the graph. Use your graph to complete the table.

x	y
0	0
2	1
4	4
6	9

x	y
3	
5	
8	

6a) Without drawing the graph give any information that you can about the graph of
$\{(x,y),\ x^2 + y^2 = 25,\ x,y \in R\}$

b) Complete the table and sketch the graph of $y = 2x^2 - 5$, $x,y \quad R$.

x	y
3	
2	
1	
0	
-1	
-2	
-3	

Chapter 5 Test NOTES AND ANSWERS

1a)

x	y
1	1
2	4
3	7
4	10

b) Domain $\{1, 2, 3, 4\}$

c) Range $\{1, 4, 7, 10\}$

d) Graph

2a) graphs

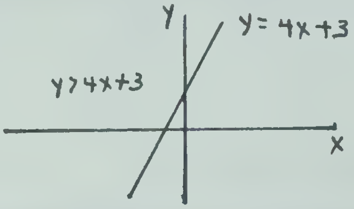
b) All have the same direction, are parallel or similar comment.

3a) $y = \frac{-4x}{3} + 4$

b) Any relation of form $y = \frac{-4x}{3} + b$

c) Any relation of form $y = mx + 4$

4a)



5. Values taken from $y = \frac{1}{4}x^2$ hence (3, 2¼), (5, 8¼), (8, 16)

Accept any reasonable values.

6a) Graph will be curved solid line

some will recognize it as a circle

some could give centre and/or radius

suggest bonus marks

b)

x	y
3	13
2	3
1	-3
0	-5
-1	-3
-2	3
-3	13

+ graph

Chapter 6 Test Part A

1. Solve:

a) $2x + 3 = 7$ b) $\frac{x}{4} - 3 = -1$ c) $3(x-2)=6$ d) $2x-(4x+1) = 3$

e) $\frac{x+1}{3} + \frac{x-1}{2} = 9$ f) $\frac{x}{2} - \frac{x=1}{5} =4$ g) $2(a+1) + 3 = 5(a-2)$

2. Solve and check

a) $(m + 3)(m-2) = m^2+4m$ b) $\frac{2a + 1}{3} - \frac{a + 1}{4} = 3$

3. Write the following work statements as equations and solve them.

a) Twice a number take away 4 is -10.

b) When the sum of a number and 7 is divided by e the result is 4.

c) A number minus 4 is equal to one half the number plus 2.

Part B

1. Solve graphically.

a) $y=3x+4$
$y=-4x-3$

b) $x+y=5$
$3x+2y=12$

2. Solve algebraically and check.

a) $y=3x+11$
$y=-2x-4$

b) $x+4y=14$
$2x-4y=4$

3. Solve.

$\frac{x}{2} - \frac{y}{5} = -4$

4. The difference between two numbers is 5. Twice the larger plus three times the smaller is 45. Find the numbers.

5. The school parking lot has three times as many cars as motorcycles. The total number of wheels was 112. (No sidecars). How many of each type of vehicle are there?

Chapter 6 Test and Notes

The test is given in two parts. Part A covers equations in one variable, part B covers equations in two variables.

Answers Part A

1. a) 2, b) 8, C) 4, d) -2, e) 11, F) 14. g) 5.

2. a) m=-2, b)a=7

3. a) -3, b) 5, c) 12.

Part B

1 a) (-1,1), b) (2,3)

2 a) (-3,2), b) (6,2)

3 (-4, 10). 4 12 and 7. 5 8 and 24.

Chapter 7 Test

1. Simplify the following ratios:

   a) 6:8 b) 8:16:12 c) 2.5:4 d) $\frac{15}{20}$ e) $\frac{1\frac{1}{3}}{2}$

2. Find the missing proportionals:

   a) $\frac{2}{3} = \frac{6}{x}$ b) $\frac{5}{7} = \frac{x}{35}$ c) 9:27 = 3:x d) 4:5:7 = 20:x:y

3. Find the mean proportionals between 4 and 9.

4. $4500 is to be divided between two partners in the ratio of 4:5
   How much will each partner receive?

5. The numbers of teeth on two gear wheels is in the ratio of 2:3
   If the smaller wheel has 30 teeth, how many has the larger wheel?

6. If 'a' varies directly as 'b'

   a) Write the variation statement.

   b) Write the variation equation.

   c) If b = 12 when a = 9 find the value for k, the variation constant.

   d) Find the value for b when a = 20

7. The time required to drain a container varies inversely as the area of the drainage hole. For a particular liquid a 250 L drum drains through a 20 cm^2 hole in 300 s.

   a) Complete the table

A (cm^2)	5	10	20	30	50
t (s)			300		

   b) Graph the data (use 100 cm^2 and 5 s as units on the axes)

   c) From the graph find 't' when 'A' = 40 s.

8. The cost for a banquet hall and catered reception is $250 for the hall with music plus $12 per person attending. What would the cost be for a reception with a) 110 guests

   b) 250 guests.

Chapter 7 Test Notes and Solutions

It is suggested that the work on the investigation, if done, be included in the evaluation.

Solutions

1a) 3:4 b) 2:4:3 c) 5:8 d) $\frac{3}{4}$ e) $\frac{2}{3}$

2a) $x = 9$ b) $x = 25$ c) $x = 9$ c) $x = 25$, $y = 35$

3) +6, -6

4) $2000, $2500

5) 45 teeth

6a) $a \propto b$ b) $a = kb$ (k constant) c) k = 3/4 of 0.75 d) 15

7a)

A	5	10	20	30	50
t	1200	600	300	200	120

c) t = 150 s

8a) $1570 b) $3250

Chapter 8 Test

1. $500 is borrowed for 9 months at 12% p.a.
   a) What interest is charged?
   b) What amount must be repaid?
2. A credit card bill has $265 carried over for 1 month. The service charge is $4.77
   a) What is the monthly interest rate charged?
   b) What is the equivalent annual rate?
3. A colour T.V. costs $495 cash or 24 monthly payments of $24.50 Find the effective annual interest rate.
4. List three sources of credit and an item that you might purchase using each source.
5. You are considering the purchase of a car for $7500, all extras included. You have been offered a trade-in allowance of $1600 on your old car. Sales tax of 7% on the difference must be paid. If interest at 18% is being charged, the monthly payment factor for 30 payments is 0.041 639 2. Find the monthly payment.
6. The monthly payment factor for 24 months at 18% is 0.049 924 1, for 36 months it is 0.036 152 3.

   If you are financing $5000 a) What is the monthly payment for 24 months and for 36 months?

   b) What is the total amount repaid over each period?

Chapter 8 Test Notes and Answers

The duties of a treasurer and the repayment schedules have not been covered in this test.

Answers

1a) \$45 b) \$545

2a) 1.8% per month b) 21.6% p.a.

3. 18%

4. Trust Company, Bank, Credit Union, Store financing; house, car, motorcycle, T.V. etc.

5. \$262.86

6a) \$249.62, \$180.76 b) \$5990.88, \$6507.36

Test Chapter 9

1. Give the dimensions of the following matrices

a) $(2\ 4\ 3)$ b) $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$ c) $\begin{pmatrix} 2 & 1 \\ 3 & 8 \end{pmatrix}$ d) $\begin{pmatrix} 4 & 6 & 3 & 5 \\ 9 & 7 & 4 & 0 \end{pmatrix}$

2. Simplify

a) $\begin{pmatrix} 3 & 4 & 7 \\ 2 & 1 & 5 \end{pmatrix} + \begin{pmatrix} 2 & -3 & 1 \\ -6 & 5 & 4 \end{pmatrix}$ b) $\begin{pmatrix} 9 & 5 \\ 6 & -2 \end{pmatrix} - \begin{pmatrix} 3 & -4 \\ 2 & 1 \end{pmatrix}$

c) $4\begin{pmatrix} 6 & 5 \\ -3 & 0 \end{pmatrix}$ d) $2\begin{pmatrix} 5 & 2 \\ 1 & 4 \end{pmatrix} - 3\begin{pmatrix} 2 & -1 \\ 6 & 3 \end{pmatrix}$

3. Little League Kings had the following record for hockey games over the season:

Dec			Jan			Feb		
Won	Lost	Tied	Won	Lost	Tied	Won	Lost	Tied
3	2	1	5	1	0	3	1	1

a) Use matrix addition to find their record for the season.

b) If a win is awarded 2 points, a loss 0 points, and a tie

1 point use matrix multiplication to find their points for the season.

4. Find the product if possible.

a) $\begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ b) $\begin{pmatrix} 5 & 2 \\ 6 & 4 \end{pmatrix} \times \begin{pmatrix} 3 & 1 & 7 \end{pmatrix}$

5. A store has three cashiers - Alice, Betty and Charles. At the end of the day their cash is described by the following matrix

$$\begin{array}{c} \\ \text{A.} \\ \text{B.} \\ \text{C.} \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{ccccc} \$1 & \$2 & \$5 & \$10 & \$20 \end{array} \\ \begin{pmatrix} 10 & 4 & 7 & 12 & 8 \\ 12 & 3 & 9 & 17 & 12 \\ 9 & 5 & 8 & 15 & 10 \end{pmatrix} \\ \\ \\ \end{array} \begin{pmatrix} 1 \\ 2 \\ 5 \\ 10 \\ 20 \end{pmatrix}$$

a) Find the total cash held in bills by each cashier as a row matrix.

b) Find the total cash in bills.

Test Chapter 9 Notes and Answers

The mechanical part of this test might be expanded if you feel the students are in need of extra marks.

Answers

1a) 1 x 3 b) 2 x 1 c) 2 x 2 d) 2 x 4

2a) $\begin{pmatrix} 5 & 1 & 8 \\ -4 & 6 & 9 \end{pmatrix}$ b) $\begin{pmatrix} 6 & 9 \\ 4 & -3 \end{pmatrix}$ c) $\begin{pmatrix} 24 & 20 \\ -12 & 0 \end{pmatrix}$ d) $\begin{pmatrix} 4 & 7 \\ -16 & -1 \end{pmatrix}$

3a) (11 4 2) 24 points

4a) (23 7) b) not possible

5a) (\$333 \$473 \$409)

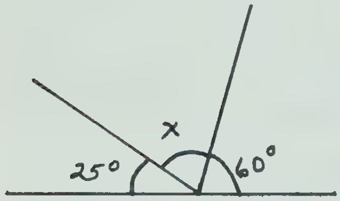
b) \$1215

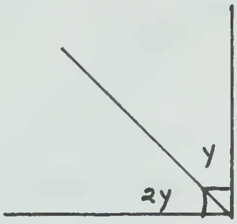
Test Chapter 10

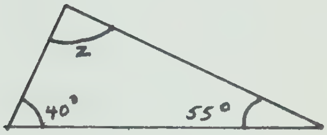
1. Change the following angle measures from revolutions to degrees.

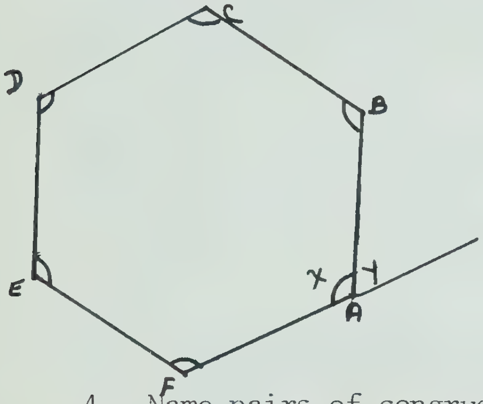
   a) ½ r b) 2.5 r c) 0.35 r d) 1/3 r

2. Solve for x, y and z in degrees:









If ABCDEF is a regular hexagon

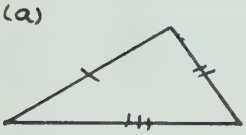
a) Find the total of the interior angles.

b) Calculate the size of each equal angle x.

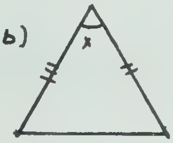
c) Calculate the size of the exterior angle y.

4. Name pairs of congruent triangles from the following set, giving you authority. (SAS, ASA, SSS)

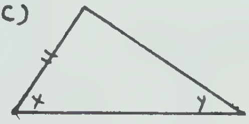
(a)



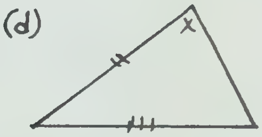
(b)



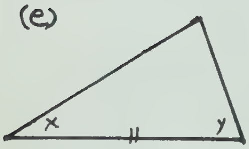
(c)



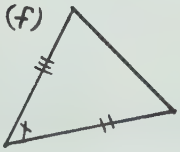
(d)



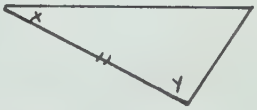
(e)



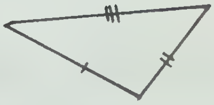
(f)



(g)

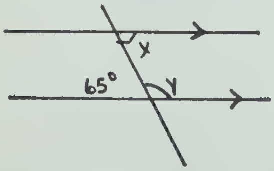


(h)

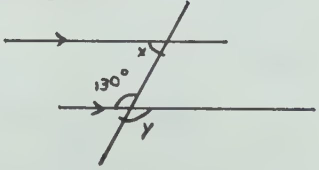


5. Solve for x, y and z in degrees:

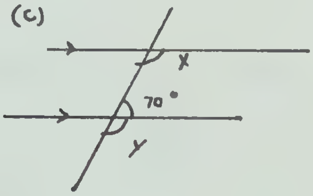
(a)

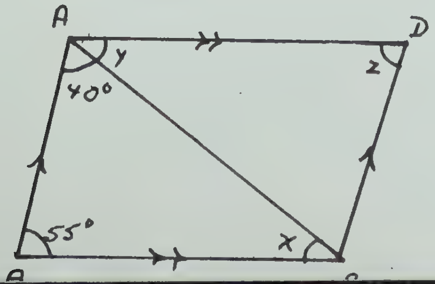


(b)

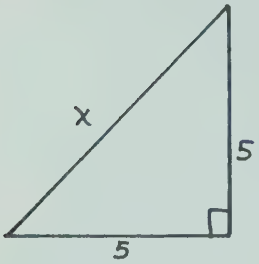


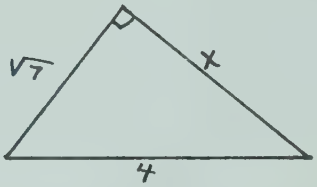
(c)

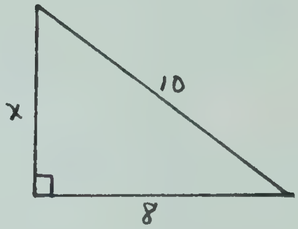




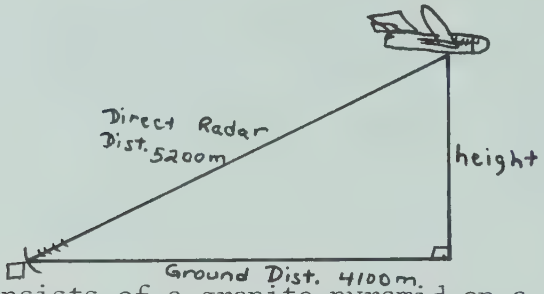
6. Find the length of the missing side of each of the following triangles. Leave irrational answers in radical form:







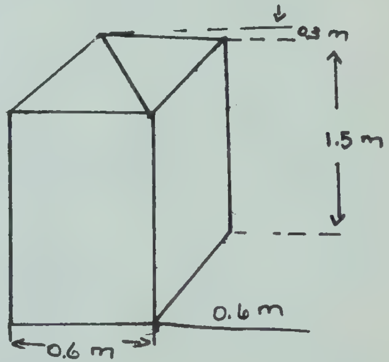
7. To the nearest 100 m find the height of the aircraft.



8a) A monument consists of a granite pyramid on a rectangular solid pedestal. Find the volume of granite.

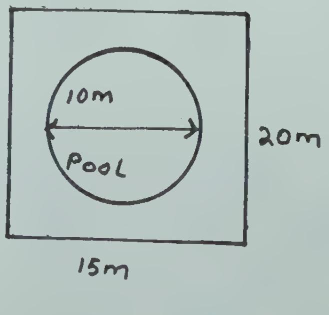
$v = l \times w \times h$

$v = \frac{1}{3}$ (area base x height)



b) To save energy (mostly theirs) the Restwells have surfaced their back yard, except for the swimming pool area, in Astroturf. What area of Astroturf was used?

$\pi = 3.14$



Chapter 10 Test Notes and Answers

This test has been kept largely numerical in nature with a straight identification question on congruent triangles. If you have stressed the investigation of congruency you will want to add more challenge to this part of the test. Question 7 can be simplified by changing the triangle to a 3-4-5 multiple if finding square root is a problem.

Answers

1a) 90° b) 900° c) 126° d) 120°

2. $x = 95^\circ$, $y = 30^\circ$, $z = 85^\circ$

3a) 720°, b) 120° , c) 60°

4a) $\cong$ (h) SSS (b) $\cong$ (f) SAS (e) $\cong$ (q) ASA

5a) $x = 65^\circ$, $y = 115^\circ$, b) $x = 50^\circ$, $y = 130^\circ$

c) $x = 110^\circ$, $y = 110^\circ$, d) $x = y = 85^\circ$, $z = 55^\circ$

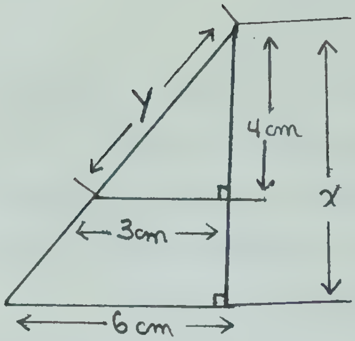
6a) $x = 5\sqrt{2}$ b) $x = 3$ c) $x = 6$

7. $h = 3200$ m

8a) $0.576 m^3$ b) $221.5 m^2$

Chapter 11 Test

1. Solve for x and y



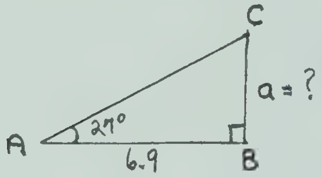
Use tables to find the following data:

2a) Sin 35° = ?
b) Cos 43° = ?
c) tan 87° = ?

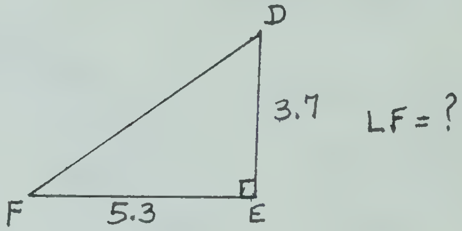
3a) Sin A = 0.9135 A = ?
b) Cos B = 0.6561 B = ?
c) tan C = 1.9626 C = ?

4. Find the required sides and angles.

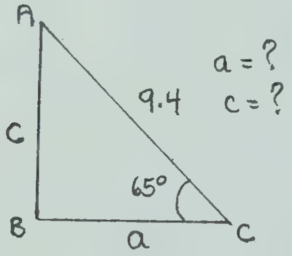
a)



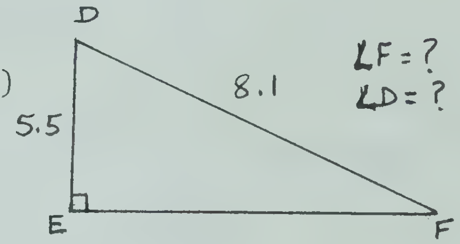
b)



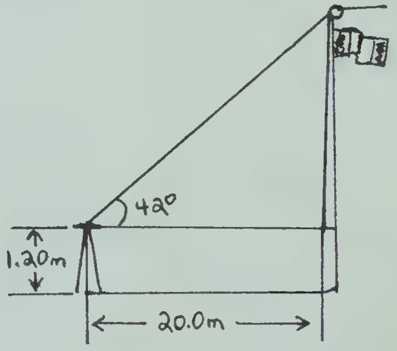
c)



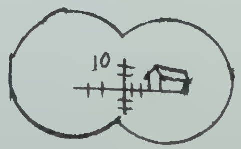
d)

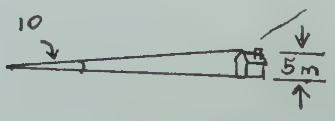


5. The angle of elevation to the top of a flag pole is 42°. The distance to the pole is 20.00m. The instrument height is 1.20m. Find the height of the pole.



6. The graticular lines in a pair of binoculars show the angles of elevation of a house to be 1°. If the house is about 5m high how far away is it to the nearest 10m?





Chapter 11 Test   Notes and Answers

Sections 11.1 to 11.5 are covered in the test. The last sections might best be tested by assignments.

Answers

1a) $x = 8\text{cm}$, $y = 5\text{cm}$

2a) 0.5736   b) 0.7314   c) 19.08

3a) 66° b) 49° c) 63°

4a) $a = 3.5$ b) $< f = 35^{\circ}$ c) $a = 4.0$, $c = 8.5$

d) $< d = 47^{\circ}$, $< f = 43^{\circ}$

5. 19.2 m

6. 290 m

Chapter 12 Test

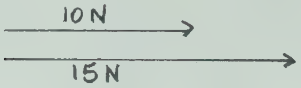
1. Mark each of the following quantities as vectors or scalars:

a) 100 km/h north b) 1500 rev/min c) 14.6 cm

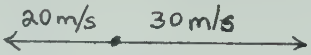
d) 50 N up

2. Give the sum of the following vectors - magnitude and direction:

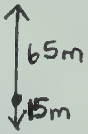
a)



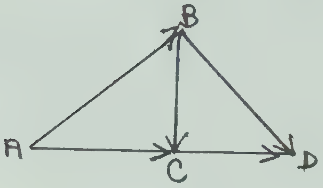
b)



c)



3.



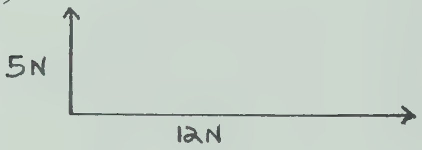
a) Give AC as a sum of two vectors.

b) Give BD as a sum of two vectors.

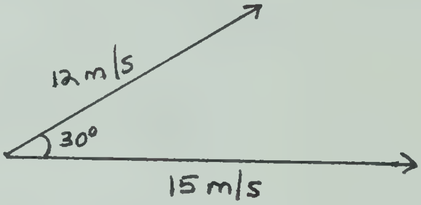
c) Give AD as a sum of three vectors.

4. Draw a scale diagram and find the magnitude and direction of the sum of the following sets of vectors:

a)



b)



5. An aircraft flies northwest at 150 km. A wind blows from the east at 30 km. Find the direction of flight and ground speed.

Chapter 12 Test Notes and Answers

The test covers section 12.1 to 12.3. The students will require rulers and protractors.

Answers

1a) vector b) scalar c) scalar d) vector

2a) 25 N right b) 10 m/s right c) 50 m up

3a) $\overline{AB} + \overline{BC}$ b) $\overline{BC} + \overline{CD}$ c) $\overline{AB} + \overline{BC} + \overline{CD}$

(If negative vectors are used other answers are possible)

4a) 13 N, 23° to the 12 N force or 67° to the 5 N force, ± 1°

b) 26m/s, 13° to 15 N force or 17° to 12 N force, ± 1°

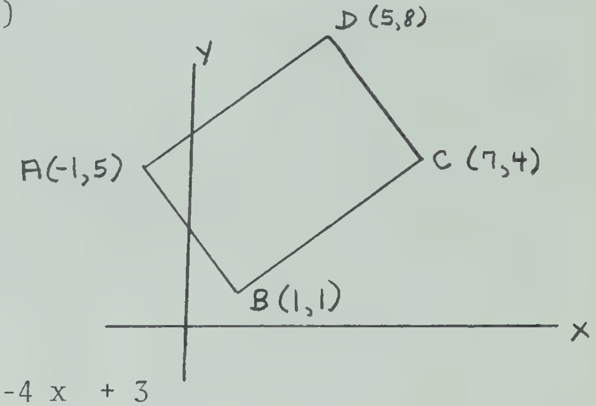
5. 172 km, N 52° East or 7° East of their heading, ± 1°, ± 2 km.

Chapter 13 Test

1. Find the slope of the following line segments:
   a) A(3,4), B(7,6) b) C(-5,2), D(0,4)
   c) E(6,4), F(6,-2) d) G(4,-3) H(2,-3)

2. Given A(2,5) and B(-1,8) select the point which is collinear with these two.
   C(4,6), D(6,1), E(-3,4), F(0,0)

3. Given figure ABCD
   a) prove AB//CD using slopes.
   b) prove $AB \perp BC$ using slopes.
   c) prove AC = BD by finding the length of each segment.



4. (1) $3x + 4y = 12$ (2) $y = \frac{-4}{3}x + 3$

   (3) $y = \frac{-3x}{4} + 7$ (4) $y = \frac{4}{3}x - 1$

   (a) Which pair of equations represent parallel lines?
   (b) Which pair of equations represent perendicular lines?

5. Find the equations of the following lines:
   (a) Through (4,5), parallel to the x axis.
   (b) y - intercept -2, parallel to $y = \frac{3}{4} + 4$.
   (c) Through (0,5), perpendicular to $y + 6x - 3$.

6. Graph the following lines on one set of axis.
   (a) Through (2,4), slope ½.
   (b) x-intercept 5, slope $-\frac{3}{4}$
   (c) Through (-2,0), slope undefined.

7. Find the lengths of the sides of ABC and tell what type of triangle it is.
   A(2,5), B(5,1), C(1,-2)

8. In an experiment different quantities of an unknown material are weighed in a container.

Mass (g)		22	28		40
Volume (cm^3)	0	8	12	16	20

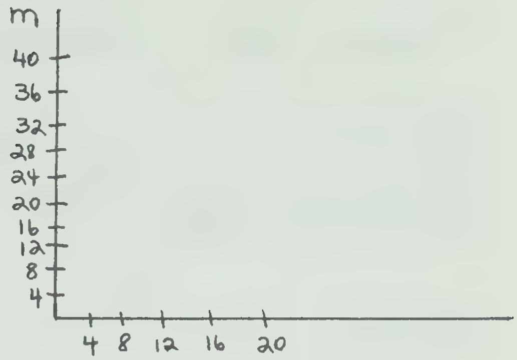
a) Graph the data (let square = 4 units)

b) What is the mass when V = 16 cm^3?

c) What is the mass of the container?

d) What is the slope of the line?

e) What is the density in g/cm^3?



Chapter 13 Test   Notes and Answers

The test includes a question or applications which makes the test a bit long for slow workers. If time is a problem the last question could be given as a home assignment.

Answers

1a) $\frac{1}{2}$ b) $\frac{2}{5}$ c) undefined   d) 0

2. $D(6,1)$

3a) Slopes equal -2 b) $-2 \times \frac{1}{2} = -1$ c) $AC = BD = \sqrt{65}$

4a) $(1)//(3)$ b) $(1)\perp(4)$ or $(3)\perp(4)$

5a) $y = 5$ b) $y = \frac{3x}{4} - 2$ c) $y = \frac{-1}{6} + 5$

6. Graphs

7. $AB = \sqrt{25} = 5$, $AC = \sqrt{50} = 5\sqrt{2}$, $BC = \sqrt{25} = 5$ right angled isosceles

8a) Graph   b) 34g   c) 10 g (when $V = 0$)
d) $\frac{3}{2}$ e) $\frac{3}{2}$ or 1.5 g/cm^3.

McGRAW·HILL RYERSO

ISBN 0-07-092436-8

QA 39-2 A12 1976 BK-1 TCH-GD-
AMT APPLIED MATHEMATICS FOR
TODAY/ /2ND ED --

39496874 CURR

000009479791

DATE DUE SLIP

RETURN JUN 12 [illegible]

RETURN FEB 10 '9[illegible]

F255

JAN 24 1984

RECOMMENDED FOR USE
IN ALBERTA SCHOOLS

QA 39.2 A12 1976 bk.1 tch.gd.
AMT :

39496874 CURR

B17039